Empirical Economics (1994)-19:659-673



# A Multivariate Time Series Analysis of the United States Aggregate Production Function

# W. DOUGLAS MCMILLIN AND DAVID J. SMYTH<sup>1</sup>

Department of Economics, Louisiana State University, Baton Rouge, LA 70803-6306, USA

Abstract: This study examines the effects of hours of work per unit of private sector capital, the relative price of energy, government capital per unit of private sector capital, and inflation on private sector output per unit of capital in the U.S. over the period 1952–90. A small vector autoregressive model that comprises the variables typically employed in single-equation estimates of the aggregate production function is used. Variance decompositions and cumulative impulse response functions indicate that hours of work per unit of private sector capital, the relative price of energy, and the inflation rate have significant effects on private sector output per unit of capital over the 1952–90 period. However, there is no evidence of a significant effect for government capital per unit of private capital. An historical decomposition that begins in 1973 with the emergence of a "productivity slump" and continues through 1990 indicates that shocks to hours of work per unit of capital but shocks to government capital are not important.

JEL Classification System-Numbers: E10, E31

### I Introduction

Early studies of the aggregate production function estimated relationships between output and inputs of labor and capital and disembodied technical progress. Typically the labor input was measured by total hours worked, the capital input by the private capital stock (usually adjusted by its utilization rate), and disembodied technical progress was approximated by a time trend. In recent years the aggregate production function has received a great deal of empirical scrutiny stimulated in part by an apparent decline in productivity growth starting about 1973. Production function researchers have developed specific hypotheses about the effect on output of energy prices, public capital stock, and inflation and have estimated production functions with extended specifications.

The ratio of the price of energy to the price of output fluctuated markedly after 1973. There was a big increase from 1973 to 1981 and a decline since then.

0377-7332/94/4/659-673 \$2.50 © 1994 Physica-Verlag, Heidelberg

<sup>&</sup>lt;sup>1</sup> The authors thank two anonymous referees and Thomas R. Beard and James S. Fackler for helpful comments. David J. Smyth acknowledges research support from the Center for Energy Studies, Louisiana State University.

Rasche and Tatom (1977, 1981) and other researchers responded by including energy as an input in the production function. Application of the first order condition made the relative price of energy a variable in the production function. Empirical studies found a significant negative relationship between output and the relative price of energy.<sup>2</sup>

The public stock of capital has both direct and indirect effects on private sector output. Direct effects arise because public capital provides intermediate services to the private sector, indirect effects stem from complementarity between government and private capital. Some studies found either the sum of federal nonmilitary and state and local government capital or just the state and local capital stock to have a positive and significant effect on private output. Recent studies have found no significant relationships.<sup>3</sup>

Inflation distorts price signals. This affects the ability of economic agents to plan as they waste time, effort, and resources trying to decipher price signals. Thus inflation may have deleterious effects on output. A negative supply side relation between inflation and output is supported by studies using production function and other approaches.<sup>4</sup>

The aim of the present study is to examine the effects of the ratio of total hours worked to private capital, the relative price of energy, government capital relative to private capital, and inflation on private sector output per unit of private capital. While earlier studies made single equation estimates of the aggregate production function, the analysis for the present paper is conducted within a vector autoregressive (VAR) model. To maintain comparability with earlier work, we use variables included in one or more of the aggregate production functions of Ratner (1983), Ram and Ramsey (1989), Tatom (1991) and Smyth (1993). The variables in the model are: (1) private sector output divided by the product of the private sector capital stock and the capacity utilization rate (OUTPUT/CAPITAL); (2) total business sector hours divided by the product of the private capital stock and the capacity utilization rate (HOURS/CAPITAL); (3) the relative price of energy (ENERGY); (4) the government capital stock divided by the private capital stock (SLCAPITAL or GOVCAPITAL depending on the specification); and (5) the inflation rate (INF). These variables

<sup>&</sup>lt;sup>2</sup> For information on the magnitude and form of the effect of the relative price of energy in production functions, see Hickman (1987), Tatom (1988, 1991), and Smyth (1993). The importance of energy prices for aggregate economic performance has been documented in several other types of studies, for example, Hamilton (1983).

<sup>&</sup>lt;sup>3</sup> Studies by Ratner (1983), Ram and Ramsey (1989), Aschauer (1989, 1993), and Munnell (1990) found significant relationships. Rubin (1991) condludes that the effect of government capital on private sector productivity is likely spurious. Tatom (1991, 1993), Smyth (1993, 1994), and Evans and Karras (1994) found that government capital has no significant effect on private sector output.

<sup>&</sup>lt;sup>4</sup> The harmful effects of inflation on output are discussed in Howitt (1990). Smyth (1993, 1994) adds inflation variables to an aggregate production function and finds significant negative relationships. For a survey of the empirical evidence, see Smyth and Schaling (1993).



Fig. 1: Model variables

are plotted in Figure 1. The VAR model is estimated using annual data for the period 1952 to 1990.<sup>5</sup>

The effects of HOURS/CAPITAL, ENERGY, SLCAPITAL (GOVCAPI-TAL), and INF are examined by the computation of variance decompositions and cumulative impulse response functions. The significance of the effects of these variables is evaluated by the calculation of standard errors for the variance decompositions and cumulative impulse response functions from Monte Carlo simulations. Additionally, an historical decomposition is computed for the 1973 to 1990 period. The historical decomposition measures the effects of the variables of interest over a particular part of the sample. The year 1973 is chosen as the starting point for the historical decomposition because of the poor performance of aggregate productivity after this point.

An advantage of the VAR approach over the single-equation approach is that, in the specification and estimation of the VAR, all variables are treated as jointly determined. At this stage, no a priori assumptions are made about exogeneity of any of the variables in the VAR. Few restrictions are placed on the way that the variables in the system interact, and the effects of one variable on another are not restricted to contemporaneous effects as is the case in the singleequation studies. However, in the computation of the variance decompositions, impulse response functions, and historical decompositions, some identifying assumptions must be made. The identifying restrictions employed here are similar to those used in single-equation estimates of production functions and thus facilitate comparison of the VAR results to the single-equation results.

Section II discusses the data and the specification of the model; the empirical results are presented and analyzed in section III. Our conclusions are summarized in section IV.

## **II** Data Description and Model Specification

The VAR model is estimated with annual data from 1952 to 1990. Observations for 1950 and 1951 are used as pre-sample data to generate the lags for the VAR. Annual data are used since the capital stock data are available only annually.

<sup>&</sup>lt;sup>5</sup> The single-equation studies typically employ a Cobb-Douglas production function and assume constant returns to scale. The assumption of constant returns to scale accounts for the form of the output, total hours, and government capital variables. Additionally, since it is assumed in these studies that the same utilization rate is applicable for both government and private capital, the capacity utilization rate does not appear in the government capital variables. However, Tatom (1991) points out that it is not necessary to assume the same utilization rate for public and private capital in order to obtain the specification used in the single-equation studies if proportionality in the use of public capital relative to private capital is assumed.

Private sector output is measured by the index of business sector output and total hours by the index of total hours in the private business sector. The relative price of energy is measured by the ratio of the producer price index for fuels and related products and the price of business sector output (the implicit price deflator for the business sector). The inflation rate is the logarithmic difference between the end of year consumer price index (all items, urban consumers) for years t and t - 1. The value of the index for the end of year t is the average of the index for December of year t and January of year t + 1.6 The capacity utilization rate is the rate for the manufacturing sector produced by the Federal Reserve Board. All the capital stock series are expressed in constant dollars. The private capital stock is the net stock of private fixed capital. Two alternative measures are used for the government capital stock. One, GOVCAPITAL, is the sum of the net stocks of federal non-military fixed capital and state and local fixed capital. The second measure, SLCAPITAL, is just the net stock of state and local fixed capital. We provide more detailed results for the state and local measure as in Ram and Ramsey (1989) state and local capital stock was significant, but federal non-military capital stock was not. Following most production function studies, the private and public sector capital stock measures for year t are the stocks at the end of year t - 1.

The capital stock data were supplied by John Musgrave of the U.S. Department of Commerce. All other data were obtained from Citibase. The data incorporate the 1991 revisions to the national accounts and capital stocks.

Although it is common to perform unit root and cointegration tests prior to specification and estimation of VAR models, these tests are not uncontroversial. Accordingly, we followed Hamilton's (forthcoming) suggestion and estimated the model with both levels and first differences of the variables.<sup>7</sup> The levels model was estimated with the log levels of OUTPUT/CAPITAL, HOURS/ CAPITAL, and SLCAPITAL (GOVCAPITAL) and INF. ENERGY was measured as the log of the producer price index for fuels and related products minus the log of the implicit price deflator for business output. To account for apparent nonstationarity, a trend term was added to each equation in the levels model. The first differences model employed the first differences of the variables in the levels model.<sup>8</sup> The optimal lag length for the VAR models was determined

<sup>8</sup> The results were unchanged when trend and trend squared were added to each equation in the levels models and when a trend term was added to each equation in the first differences models.

<sup>&</sup>lt;sup>6</sup> The variance decomposition results reported later in the text were unaltered when the inflation rate was computed from an index whose value for year t was set equal to the value in December of year t.

<sup>&</sup>lt;sup>7</sup> The ability of the unit root tests to distinguish between trend and difference stationarity has been challenged by Sims (1988) and DeJong and Whiteman (1991), among others. Furthermore, DeJong (1992) has recently questioned the ability of cointegration tests to correctly identify cointegration among the series. However, Phillips (1991) has strongly criticized the methodology and conclusions of these studies.

by a sequence of likelihood ratio tests and was found to be 2 years.<sup>9</sup> Q-statistics indicated no serial correlation in the residuals of the models.

#### **III** Empirical Results

As noted in section I, the effects of HOURS/CAPITAL, INF, ENERGY, and SLCAPITAL (GOVCAPITAL) are evaluated by computing variance decompositions, cumulative impulse response functions, and historical decompositions. These measures are based on the moving average representation of the VAR model and hence reflect both direct and indirect effects. The variance decompositions show the percent of the forecast error variance for each variable that can be attributed to its own innovations and to shocks to the other variables in the system. The variance decomposition for OUTPUT/CAPITAL thus indicates the percent of its forecast error variance accounted for by shocks to itself, HOURS/CAPITAL, SLCAPITAL (GOVCAPITAL), ENERGY, and INF. If a variable is an important determinant of movements in OUTPUT/CAPITAL, it should explain a significant portion of the forecast error variance in OUT-PUT/CAPITAL. As noted by Sims (1982), if a variable explains a large and significant portion of the forecast error variance of OUTPUT/CAPITAL, this could be interpreted as a strong Granger-causal relation.

The cumulative impulse response functions indicate the size and direction of effect of a one-standard deviation shock to a variable on the other variables in the system. By computing cumulative impulse response functions, the direction of effect of a shock to HOURS/CAPITAL, SLCAPITAL (GOVCAPITAL), ENERGY, and INF on the level of OUTPUT/CAPITAL can be determined. The value of the cumulative impulse response function in any period is the sum of the effect of shocks to a variable on another variable in the current and prior periods.

In order to determine the significance of the effects measured by the variance decompositions and cumulative impulse response functions, standard errors are estimated from Monte Carlo simulations that employ 1000 draws. For the vari-

<sup>&</sup>lt;sup>9</sup> A maximum lag of 4 was considered and the likelihood ratio tests employed the small sample correction suggested by Sims. The testing began with a comparison of a 4 lag model with a 3 lag model. If the null hypothesis that the fourth lags were equal to zero could not be rejected, the 3 lag model was tested against a 2 lag model. This continued until the null hypothesis was rejected.

Since the maximum lag was 4, estimation for the likelihood ratio tests was done for 1954–90. Since the optimal lag was 2, the estimation of the model whose results are reported in the text was done for 1952–90 in order to provide as many degrees of freedom as possible. However, the variance decomposition results for a model estimated over 1954–90 are essentially identical to those in Table 1.

ance decompositions, the point estimates of the proportion of the forecast error variance explained by each variable are judged to be significant if the point estimate is at least twice the estimated standard error. For the cumulative impulse response functions, confidence intervals for the point estimates are provided by constructing a two standard deviation band around the point estimates.

Since the equations of the VAR contain only lagged values of the system's variables, any contemporaneous relations among the variable are reflected in the correlation of the residuals across equations. The cross-equation residual correlation is removed by employing the Choleski decomposition. In this approach, the variables are ordered in a particular manner. When a variable higher in the order changes, variables lower in the order are assumed to change, and the extent of the change depends upon the covariance of the variables higher in the order with those lower in the order. Thus, credit for any correlation between two variables is assigned to the variable higher in the ordering. In this manner some economic structure is imposed in the computation of the variance decompositions, cumulative impulse response functions, and historical decompositions.<sup>10</sup>

The ordering employed is ENERGY, INF, SLCAPITAL (GOVCAPITAL), HOURS/CAPITAL, OUTPUT/CAPITAL. Placing OUTPUT/CAPITAL last is the most defensible ordering for this study since it is consistent with the single-equation studies cited earlier. As in the single-equation studies, all other variables in the model are allowed to contemporaneously alter OUTPUT/

<sup>10</sup> The variance-covariance matrix for the levels model with SLCAPITAL is:

	OUTPUT/ CAPITAL	HOURS/ CAPITAL	SLCAPITAL	INF	ENERGY
OUTPUT/ CAPITAL HOURS/	.378 × 10 <sup>-3</sup>	.94	.07	69	55
CAPITAL SLCAPITAL INF ENERGY	$.354 \times 10^{-3}$ $.005 \times 10^{-3}$ $164 \times 10^{-3}$ $658 \times 10^{-3}$	$.376 \times 10^{-3}$ 007 × 10 <sup>-3</sup> 133 × 10 <sup>-3</sup> 487 × 10 <sup>-3</sup>	$\begin{array}{c}11 \\ .011 \times 10^{-3} \\008 \times 10^{-3} \\019 \times 10^{-3} \end{array}$	$\begin{array}{c}56 \\19 \\ .150 \times 10^{-3} \\ .452 \times 10^{-3} \end{array}$	41 09 .60 $3.81 \times 10^{-3}$

The numbers above the diagonal are the correlation coefficients while the numbers below the diagonal are the covariances.

The variance-covariance matrix for the first differences model with SLCAPITAL is:

	OUTPUT/ CAPITAL	HOURS/ CAPITAL	SLCAPITAL	INF	ENERGY
OUTPUT/ CAPITAL HOURS/	$.480 \times 10^{-3}$	.94	.21	72	49
CAPITAL SLCAPITAL INF ENERGY	$.459 \times 10^{-3}$ $.019 \times 10^{-3}$ $220 \times 10^{-3}$ $760 \times 10^{-3}$	$\begin{array}{r} .497 \times 10^{-3} \\ .004 \times 10^{-3} \\212 \times 10^{-3} \\651 \times 10^{-3} \end{array}$	.04 .016 $\times$ 10 <sup>-3</sup> 003 $\times$ 10 <sup>-3</sup> 004 $\times$ 10 <sup>-3</sup>	69 06 $.191 \times 10^{-3}$ $.370 \times 10^{-3}$	42 01 .38 $4.93 \times 10^{-3}$

CAPITAL. Thus placement of OUTPUT/CAPITAL last facilitates comparison of the results of this study with those of the single-equation studies. Placement of ENERGY first is based on the assumption that contemporaneous shocks to the relative price of oil stem more from developments in the world oil market than from shocks to the other model variables. INF is placed second because it is primarily determined by prior monetary actions and not by contemporaneous movements in SLCAPITAL, HOURS/CAPITAL, or OUTPUT/CAPITAL. SLCAPITAL and HOURS/CAPITAL are placed third and fourth, respectively, although the results are essentially unchanged if HOURS/CAPITAL is placed before SLCAPITAL.<sup>11</sup>

The variance decompositions for both the levels and first differences models are reported in Table 1. Because the focus of the paper is on explaining the behavior of OUTPUT/CAPITAL, only the variance decomposition results for this variable are reported. The estimated standard errors are in parentheses below the point estimates. A \* indicates the point estimate is at least twice the standard error. Variance decompositions at horizons of 1, 2, 4, and 8 years are reported in order to convey a sense of the dynamics of the system. Estimates for the models with SLCAPITAL are reported in part A of the table while part B reports results for the models with GOVCAPITAL. We observe significant effects of ENERGY, INF, and HOURS/CAPITAL on OUTPUT/CAPITAL for both the levels and first differences models. Jointly ENERGY, INF, and HOURS/CAPITAL explain at least 84% of the forecast error variance in OUT-PUT/CAPITAL in all the models. However, neither the effects of SLCAPITAL nor the effects of GOVCAPITAL are significant over any horizon. The results for the first differences models are generally within one standard deviation of those for the levels models, and are, in all cases, within two standard deviations. The main quantitative difference between the levels and first differences models is that HOURS/CAPITAL has the largest effects in the levels model while in the first differences models, the magnitude of HOURS/CAPITAL's effects falls somewhat and is approximately the same magnitude as the effects of ENERGY. However, the models indicate the same pattern of significance for ENERGY, INF, and HOURS/CAPITAL and lack of significance for SLCAPITAL and GOVCAPITAL.

The cumulative impulse response functions are reported in Figure 2. In order to conserve space, only the cumulative impulse response functions for the SLCAPITAL levels model are reported. Cumulative impulse response functions for the other models are similar. We observe that shocks to ENERGY have

<sup>&</sup>lt;sup>11</sup> An alternative to the Choleski decomposition is the structural VAR approach of Bernanke (1986). Bernanke suggests specifying and estimating a structural model using the residuals from the VAR. The residuals of the structural model are purged of contemporaneous correlation. This procedure is not employed because of the nature of the VAR estimated. Since the only variables included are variables employed in single-equation estimates of the aggregate production function, it would be difficult to specify an acceptable structural model. For the same reason, the procedure of Blanchard and Quah (1989) is not used.

A. MOUCIS WILL STATC ALIU	local capital								
Variable	Horizon	Explained b	y Shocks to			Explained by	y Shocks to		
		Levels Mod	G			First Differe	nces Model		
	1	ENERGY	INF	SLCAPITAL	HOURS/	ENERGY	INF	SLCAPITAL	HOURS/
OUTPUT/CAPITAL	1	30.1	20.2	0.2	44.1	24.4	33.8	2.9	32.2
		(12.3)*	*(6.6)	(2.3)	(10.7)*	(11.7)*	(11.2)*	(4.1)	(8.8)*
	2	27.6	19.1	0.4	45.3	30.3	26.0	3.5	27.4
		(11.1)*	(6.3)*	(2.9)	(10.6)*	(12.5)*	(10.2)*	(5.5)	(8.1)*
	4	24.9	26.2	6.0	39.0	27.9	33.2	3.5	24.0
		(10.2)*	(6.5)*	(2.8)	(6.8)*	(10.9)*	(10.0)*	(5.4)	(7.5)*
	00	26.0	26.1	3.9	34.3	27.3	33.8	4.2	23.6
		(10.0)*	(9.1)*	(3.7)	(6.5)*	(10.6)*	(10.0)*	(5.6)	(7.3)*
B. Models with total gove	rnment capita	al							
Variable	Horizon	Explained b	y Shocks to			Explained by	y Shocks to		
		Levels Mode	el			First Differe	nces Model		
		ENERGY	INF	GOV-	HOURS	ENERGY	INF	GOV-	HOURS/
OLITPITT/CADITAL		000	200	CALITAL	CAFILAL	15.7	202	CAFILAL	CAFIIAL
	•	(12.3)*	*(8)*	(3.6)	(10.3)*	(12.0)*	(11.3)*	(2.2)	*(2.6)
	2	26.8	19.1	2.9	43.9	29.4	23.1	0.3	32.6
		(11.4)*	*(0.6)	(4.2)	(10.0)*	(12.6)*	(10.2)*	(2.8)	(6.2)*
	4	22.7	29.5	3.0	34.9	27.3	30.6	1.4	27.9
		(10.1)*	*(6.6)	(3.8)	*(0.6)	(11.0)*	(6.8)*	(3.4)	(8.2)*
	00	25.0	28.7	7.2	29.9	26.6	31.6	2.4	27.1
		(10.0)*	*(9.6)	(4.8)	(8.6)*	(10.6)*	(10.0)*	(4.0)	(8.1)*

<sup>1</sup> Standard errors are in parentheses below the point estimates. A \* indicates the point estimate is at least twice the standard error.

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Fig. 2: Impulse response functions

persistent negative effects on the level of OUTPUT/CAPITAL over an 8 year horizon, as indicated by the fact that almost all of the two standard deviation interval estimate for a shock to ENERGY lies below zero at all horizons. If the commonly used (see Blanchard and Quah (1989)) one standard deviation interval estimate is plotted (not shown in order to save space), the entire interval is below zero. Shocks to INF have, with one exception, negative effects on OUT-PUT/CAPITAL, but the bulk of the confidence interval is clearly below zero only for the first two periods. The one standard deviation interval presents a similar picture with the exception that the interval is below zero for the first three periods. Since INF is a nominal variable, it is not surprising that shocks to INF do not have a long-lived effect on OUTPUT/CAPITAL. Shocks to SLCAPITAL produce a confidence interval for OUTPUT/CAPITAL that is almost virtually centered on zero at all horizons. The same is true for the one standard deviation interval estimate. Similar results are found for GOVCAPI-TAL. Finally, shocks to HOURS/CAPITAL have persistent positive effects on OUTPUT/CAPITAL as indicated by the fact that virtually all of the confidence interval lies well above zero at all horizons. The entire interval estimate lies above zero in the one standard deviation case.

Table 2 reports the results of historical decompositions for both the levels and first difference models which indicate the effects of ENERGY, INF, SLCAPI-TAL (GOVCAPITAL), and HOURS/CAPITAL on OUTPUT/CAPITAL over

Table 2. Historical decompositions<sup>1</sup>

	ROOT-MEA	N-SQUARED-H	ERRORS	
	SLCAPITAL	MODEL	GOVCAPITAL MODEL	
	LEVELS	DIFFER- ENCES	LEVELS	DIFFER- ENCES
BP	.02448	.02627	.02485	.02483
BP + ENERGY	.02102 (.86)	.02252 (.86)	.02172 (.87)	.02126 (.86)
BP + INF	.01917 (.78)	.02075 (.79)	.01835 (.74)	.02036 (.82)
BP + SLCAPITAL	.02417 (.99)	.02545 (.97)		
BP + GOVCAPITAL			.02495 (1.0)	.02450 (.99)
BP + HOURS/CAPITAL	.02151 (.88)	.02623 (1.0)	.02231 (.90)	.02459 (.99)
BP + ENERGY + INF	.01514 (.62)	.01518 (.58)	.01529 (.62)	.01518 (.61)
BP + ENERGY + INF + HOURS/CAPITAL	.00750 (.31)	.01061 (.40)	.00692 (.28)	.00942 (.38)
BP + ENERGY + INF + HOURS/CAPITAL + SLCAPITAL	.00728 (.30)	.00974 (.37)		Sec.
BP + ENERGY + INF + HOURS/CAPITAL + GOVCAPITAL			.00751 (.30)	.01041 (.42)

<sup>1</sup> The numbers in parentheses are the ratios of the root-mean-squared errors for the base projection (BP) plus the contribution of the shocks to a particular variable(s) to the base projection root-mean-squared error.

the 1973–90 period. As noted earlier, the starting point is the approximate start of a "productivity slump". The historical decomposition assigns credit for the difference between what can be called the base projection for a series and the actual series to shocks to variables in the system. The base projection is a forecast over the period 1973–90 which is formed using only information prior to 1973. The extent to which a series that adds the shocks to a particular variable(s) over the 1973–90 period to the base projection is closer to the actual series than is the base projection alone is a measure of the importance of that variable(s). The root-mean-squared error, RMSE, for the base projection for OUTPUT/CAPITAL is reported, as are the RMSEs for the base projection plus the contribution of the shock to each of the other variables. The ratio of the RMSE for the sum of the base projection and the contribution of each variable to the RMSE for just the base projection is given in parentheses.

The historical decomposition can be described more formally in the following way. Like the variance decompositions and impulse response functions, the historical decompositions are based upon the moving average representation of the VAR. This moving average representation can be written as:

$$X_t = \sum_{i=0}^{\infty} M_i \mu_{t-i}$$

where  $X_t = a$  column vector of the variables in the system,  $\mu_{t-i} = \text{column vector}$ of shocks to the elements of X in period t - i,  $M_i = \text{matrix}$  of impulse response weights conformable to the dimensions of X and  $\mu$ . Consider a base period which runs from observation 1 to observation T (1952–1972 in this paper). The value of X in periods subsequent to T may be written as:

$$X_{T+j} = \sum_{i=j}^{\infty} M_i \mu_{T+j-i} + \sum_{i=0}^{j-1} M_i \mu_{T+j-i}$$

where  $\sum_{i=j}^{\infty} M_i \mu_{T+j-i} =$  base projection or forecast of  $X_{T+j}$  based only on information available at time T, and  $\sum_{i=0}^{j-1} M_i \mu_{T+j-1} =$  the part of X accounted for by shocks since T. The elements of the second term on the right-hand side of the equation are used to determine the extent to which addition of the shocks to a particular variable(s) to the base projection generates a series that is closer to the actual series  $(X_{T+j})$  than is the base projection alone (first term).

In the levels model with SLCAPITAL, we observe that ENERGY reduces the base projection RMSE by about 14%, INF shocks reduce the base projection RMSE by about 22%, and shocks to HOURS/CAPITAL reduce the base projection RMSE by about 12%. Together these three variables reduce the base projection RMSE by about 70%.<sup>12</sup> We observe that shocks to SLCAPITAL

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<sup>&</sup>lt;sup>12</sup> The joint effect is not merely the sum of the individual effects since the interaction of shocks to HOURS/CAPITAL, ENERGY, and INF is not taken into account if the individual effects are summed.

When a dummy variable that takes on a value of 1 from 1973–90 and 0 elsewhere was added to each equation of the system, the variance decomposition results were essentially identical to those in Table 1.

have only very small effects, both when they alone are added to the base projection and when they are considered in conjunction with shocks to ENERGY, INF, and HOURS/CAPITAL. There thus does not appear to be a monocausal explanation of the behavior of OUTPUT/CAPITAL over the 1973–90 period. Shocks to ENERGY, INF, and HOURS/CAPITAL are clearly quite important, but shocks to government capital do not appear to be important. Similiar results are found in the levels model with GOVCAPITAL. The results for EN-ERGY, INF, and SLCAPITAL (GOVCAPITAL) are comparable for the first differences models, but the effects of HOURS/CAPITAL appear less important in the first differences models than in the levels models.

## **IV** Conclusions

This study has examined the effects of the relative price of energy, the ratio of hours of work to private capital, the ratio of government capital to private capital, and inflation on private sector output per unit of private capital over the period 1952–90. The framework for analysis is a small vector autoregressive model that comprises the variables typically employed in single-equation estimates of the aggregate production function. The model is estimated in both levels and first differences specifications. A major difference of the current study with the single-equation studies is that no assumptions about exogeneity are made in the specification and estimation of the model. However, in computing the variance decompositions, cumulative impulse response functions, and historical decompositions, the economic structure imposed reflects a desire for comparability with the single-equation studies and other theoretical considerations.

Variance decompositions and cumulative impulse response functions indicate significant effects of the relative price of energy, the inflation rate, and the ratio of hours of work to private capital on private sector output per unit of private capital over the 1952-90 period. However, there is no evidence of a significant effect of government capital, measured either by the net stock of government (federal plus state-local) non-military capital or the net stock of state-local capital, on private sector output per unit of private sector capital. An historical decomposition that begins in 1973 with the emergence of a "productivity slump" and continues through 1990 indicates that shocks to the relative price of oil, inflation, and the ratio of hours of work to private capital appear important in explaining the behavior of private sector output per unit of private capital in the levels models. In the first differences models, shocks to the relative price of oil and inflation are again important in explaining the behavior of private sector output per unit of private capital. Shocks to government capital do not appear to contribute to the "productivity slump" over this period. Our results give no support to the views of those economists who believe that it is necessary to increase the stock of government capital to stimulate productivity in the private sector.

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