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Estimating the effects of monetary policy shocks: does lag structure matter?

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This paper examines the implications of lag structure for estimating the effects of monetary policy shocks in a VAR. A symmetric lag structure in which all variables have the same lag length and an asymmetric lag structure in which the lag length differs across variables but is the same for a particular variable in each equation of the model are examined. This is important in light of the fact that the true lag structure is generally not known. Four commonly used identification schemes are employed to identify monetary policy shocks. Monte Carlo simulations strongly indicate that the lag structure of a VAR model does matter when assessing the quantitative effects of monetary policy shocks. Given the inherent uncertainty about the true lag structure in practice, it is thus important that one compare the impulse response functions from both symmetric lag and asymmetric lag VARs in assessing the effects of monetary policy shocks.

I. INTRODUCTION

A critical element of the monetary policy process is knowledge of the quantitative effects of policy actions. Vector Autoregressive (VAR) models have been widely used in recent years in estimating the effects of monetary policy shocks on the US economy. There are a number of critical issues that must be addressed prior to estimating these effects. These include determination of the dimension of the model (i.e. the variables that enter the model), the method of identifying the structural shocks, and the lag length and lag structure of the model. A great deal of effort has been focused on examining the implications of the dimension of the model, alternative methods of identifying structural shocks, and lag length for estimating the effects of structural shocks. For example, Christiano *et al*. (1994, 1996, 1998), Gordon and Leeper (1994), Lastrapes and Selgin (1995), Pagan and Robertson (1995, 1998), Leeper *et al*. (1996), Bernanke and Mihov (1998), and McMillin (2001) have examined alternative identification schemes, and Leeper *et al*. (1996) and Christiano *et al*. (1998), among others, have also considered the dimension of the model. Further, it is now common practice to determine whether results are sensitive to lag length. However, relatively little effort has been directed to examining the implications of alternative lag structures for estimating the effects of shocks in VAR models. Consequently, the aim of this paper is to use Monte Carlo simulations to explore the implications of alternative lag structures for estimating the effects of monetary policy shocks.

Traditionally, most VAR models have been estimated using symmetric lag structures in which the same lag length is used for all variables in all equations. However, the routine use of symmetric lag VARs has recently been questioned by Keating (2000) who argues that asymmetric lag VARs in which the lag length can differ across variables in the model but is the same for a particular variable in each equation of the model may be more

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appropriate.[1] It is widely recognized that symmetric lag VAR models frequently generate a large number of statistically insignificant coefficients (Runkle, 1987; Rudebusch, 1998; Keating, 2000).[2] This may be problematic in assessing the effects of shocks within the context of VAR models because the impulse responses and variance decompositions are functions of the estimated reduced-form coefficients. Keating (2000) argued that optimally selected asymmetric lag VARs will typically have a smaller number of estimated parameters than do symmetric lag VARs. Using a small structural VAR model, he found that an asymmetric lag VAR generates relatively fewer insignificant reduced-form parameters than do symmetric lag VARs and that confidence intervals for impulse response functions tended to be narrower for an asymmetric lag VAR than for a symmetric lag VAR.

There is, however, no theoretical reason to believe that either a symmetric lag structure or an asymmetric lag structure is more appropriate in most VAR models. Keating (2000) showed that an asymmetric lag structure in a VAR is theoretically possible if a structural model is characterized by asymmetric lags. However, unfortunately, very seldom does theory provide any guidance as to the appropriate type of lag structure. Since the moving average representation of a VAR model is a function of the estimated coefficients of the VAR, the type of lag structure employed may be important in the computation of impulse response functions and hence in the assessment of the effects of structural shocks. Braun and Mittnik (1993) show that the estimators of a VAR whose lag length differs from the true lag length are inconsistent as are the impulse responses and variance decompositions.

Given uncertainty about the true type of lag structure, the goal of this paper is to use Monte Carlo simulations to examine empirically the implications of symmetric and Keating-type asymmetric lag structures for the computation of the effects of monetary policy shocks. Although the earlier studies cited in the first paragraph of this paper considered alternative model variables, alternative methods of identifying policy shocks, alternative lag lengths, and alternative samples, they all employed symmetric lag VARs. Since McMillin (2001) finds, for a given symmetric lag VAR model and sample period, that the magnitude and timing of the effects of monetary policy vary to some degree across identification schemes, four widely employed identification schemes for monetary policy shocks are examined. The identification schemes are the approaches suggested by Christiano *et al.* (1994, 1996, 1998), Strongin (1995), Bernanke and Mihov (1998), and Blanchard and Quah (1989). These four schemes do not exhaust all identification schemes employed in the literature, but are among the most commonly used schemes and serve to illustrate that the results of this paper are similar across different identification procedures.

The effects of monetary shocks are evaluated by estimating and comparing impulse responses from both traditional symmetric and Keating-type asymmetric lag VARs. To investigate the distortions in the impulse response functions due to lag structure misspecification, Monte Carlo simulations are employed. In these simulations, a true lag structure is first specified, and the true impulse response functions are computed. Then, on each draw of the simulation, artificial data are generated, a lag structure different from the true structure is specified, and impulse response functions are computed. These impulse response functions are then compared to the true impulse response functions.

The rest of this paper proceeds as follows. Section II briefly discusses the alternative identification schemes and describes the empirical methodology. Section III discusses the results of the Monte Carlo simulations, and Section IV provides a summary and conclusions.

II. MODEL SPECIFICATION, IDENTIFICATION OF POLICY SHOCKS, AND DESIGN OF SIMULATIONS

Model specification

The analysis in this paper is performed within a VAR model that comprises output, the price level, a commodity price index, the federal funds rate, total reserves, and nonborrowed reserves, the variables used by Christiano *et al.* (1994, 1996, 1998) and Bernanke and Mihov (1998). All data are from the DRI Basic Economics database. The variables, with their exact description and database name in parentheses, are as follows: *output* (real chain-weighted gdp: *gdpq*), the price level (the chain-weighted price index for gdp: *gdpdfc*), commodity prices (the Commodity Research Bureau's spot market price index for all commodities: *psccom*), total reserves adjusted for reserve requirements (*fmrra*), nonborrowed reserves

[1] Hsiao (1981) first examined the possibility of asymmetric lag VAR models. His asymmetric lag VAR model differs from Keating's by allowing the lag length on each variable in each equation to differ. In the Hsiao-type asymmetric lag VAR models, an extensive iterative procedure is required to appropriately specify a lag structure which makes it virtually impossible to implement the type of Monte Carlo simulation employed in this paper. Further, as is well known, Hsiao's technique of lag length selection is often sensitive to the order in which variables are considered. See Caines *et al.* (1981), McMillin and Fackler (1984), and Keating (2000). Finally, because the specification of each equation in the model is different, ordinary least squares is not appropriate for estimating a Hsiao-type asymmetric lag VAR.

[2] Gordon and King (1982) also pointed out that VAR models usually contain only a limited number of variables since the symmetry in lags rapidly erodes the degree of freedom.

adjusted for reserve requirements (*fmrnbc*), and the federal funds rate (*fyff*). Following Christiano *et al.* (1994, 1996, 1998) and Bernanke and Mihov (1998), the logs of output, the price level, and commodity prices are used, while the level of the federal funds rate is employed. These variables are referred to from now on as *LRGDP*, *LGDPD*, *LPCOM*, and *FFR*, respectively.

However, both total reserves and nonborrowed reserves are normalized by a 12-quarter moving average of total reserves. This type of normalization rather than logs is used since the Bernanke–Mihov identification scheme is based on a linear model of the reserves market. Equilibrium in this model requires the demand for total reserves to equal the supply of total reserves. The structure of the model is based upon the fact that the supply of total reserves is the sum of nonborrowed reserves and borrowed reserves. Hence, using logarithms is not consistent with this type of linear model. Normalizing total reserves and nonborrowed reserves in this fashion is similar in spirit to both Strongin (1995) and Bernanke and Mihov (1998). Normalized total reserves and nonborrowed reserves are hereafter referred to as *TR* and *NBR*, respectively.

Identification schemes

Four identification schemes are employed; two use a pure Choleski decomposition which imposes recursive contemporaneous identifying restrictions, a third blends the Choleski decomposition with a structural model of the reserves market, and the fourth relies upon long-run restrictions to identify monetary policy shocks. Since the three schemes using contemporaneous identifying restrictions are well known, they will be presented only briefly. The scheme employed by Christiano *et al.* (hereafter CEE) uses a Choleski decomposition with the variables in the following ordering: *LRGDP*, *LGDPD*, *LPCOM*, *NBR*, *TR*, and *FFR*. *NBR*, the variable most directly controlled by the Federal Reserve, is taken as the policy variable.[3] The second identification scheme is in the spirit of Strongin (1995) who also employs *NBR* as the policy variable. Strongin's (hereafter STR) scheme imposes the following contemporaneous causal ordering: *LRGDP*, *LGDPD*, *LPCOM*, *TR*, *NBR*, and *FFR*. Note that the contemporaneous causal link between *NBR* and *TR* is reversed compared to the CEE scheme.[4] The third identification scheme considered in this paper is Bernanke and Mihov's (1998) semi-structural VAR. This scheme (hereafter BM) extracts monetary policy shocks from a model of the reserves market estimated from VAR residuals for *NBR*, *TR*, and *FFR* that are orthogonalized with respect to the other model variables.[5] The monetary policy shock is the residual from a Federal Reserve reaction function in which the shock to *NBR* is modelled as a linear function of the shock to *TR* demand and borrowed reserve demand.

The long-run restrictions approach (hereafter LR), first introduced by Blanchard and Quah (1989) and Shapiro and Watson (1988), does not impose restrictions on contemporaneous relationships among the model variables as is done in the other schemes. Instead, restrictions on the long-run relations among the variables are imposed.

[3] Although Bernanke and Blinder (1992) contend that *FFR* is a good measure of monetary policy, Eichenbaum (1992) argues that *NBR* is a preferred measure. The CEE scheme, as the ordering implies, assumes that monetary policy affects *LRGDP*, *LGDPD*, and *LPCOM* only with a lag and that the Federal Reserve has full current information on these three variables. The scheme also assumes that monetary policy has a contemporaneous effect on *TR* and *FFR*, although the Federal Reserve responds to movements in these variables only with a lag.

[4] Although Strongin constructed two different VARs with three variables and five variables, respectively, this paper employs the same six variables as CEE. However, the essential point of the Strongin scheme that shocks to *TR* reflect reserve demand shocks is maintained. In this view, *NBR* shocks are viewed as a mixture of reserve demand shocks and policy shocks. When the Federal Reserve targets *FFR*, as it did over most of sample period used here, a reserve demand shock would tend to raise *FFR* unless the Federal Reserve expanded *NBR*. Thus, orthogonalized policy shocks can be extracted by placing *TR* prior to *NBR* in ordering.

[5] Bernanke and Mihov assumed the following structural model for bank reserves:

$$\mu_{tr} = -\alpha\mu_{ffr} + v^d \tag{1}$$

$$\mu_{br} = \beta(\mu_{ffr} - \mu_{disc}) + v^b \tag{2}$$

$$\mu_{nbr} = \phi^d v^d + \phi^b v^b + v^s \tag{3}$$

where the μs represent the VAR residuals that are orthogonalized with respect to *LRGDP*, *LGDPD*, and *LPCOM*, and the vs are structural shocks. Subscripts *tr*, *ffr*, *br*, *disc* and *nbr* represent total reserves, the federal funds rate, borrowed reserves, the discount rate, and nonborrowed reserves, respectively. Thus Equation 1 describes *TR* demand, while Equation 2 describes borrowed reserve demand. Equation 3 represents the Federal Reserve's reaction function; hence v^s can be interpreted as the shock to monetary policy that we are interested in identifying. Equation 3 implies that the Federal Reserve has current information on the shocks to both *TR* and borrowed reserves. In this paper, we slightly modify the structural model, based upon Bernanke and Mihov's results and suggestions. That is, we impose the restriction that α=0 on Equation 1; the innovation in *TR* is assumed to reflect a reserve demand shock, as in Strongin. This restriction is imposed because Bernanke and Mihov pointed out that a just-identified model with α=0 performs well. Also, in Equation 2, the discount rate shocks are set to zero in order to compare the Christiano *et al.*, and Strongin schemes that do not explicitly consider the discount rate. The structural model is estimated using a two-step efficient GMM procedure (RATS procedure measure.src) provided by Bernanke and Mihov.

Assumptions about the long-run neutrality of money are used to identify monetary policy shocks in this approach. In order to implement this procedure, the model is specified as comprising *LRGDP*, the log level of real commodity prices (*LRPCOM* = *LPCOM*–*LGDPD*), *LPCOM*, *NBR*, *TR*, and *FFR*. *LGDPD* no longer enters as a separate variable, but the effect of monetary policy on *LGDPD* can be recovered from the separate effects of monetary policy on *LRPCOM* and *LPCOM*. *NBR* is assumed to be the monetary policy variable. All variables are first differenced prior to estimation, i.e. a unit root is imposed. With the model in first differences, a Choleski decomposition of the long-run relations allows imposition of the neutrality assumptions. In a VAR estimated in first difference form, the long-run effect of a shock to monetary policy on the level of model variables is the cumulative sum of the relevant part of the moving average representation. Note that in a model estimated in first differences the moving average representation indicates the effect of the shock on the changes in the variables; hence to obtain the effect on the levels of the variables, the effects on changes must be cumulated. Keating (2002) demonstrates that neutrality restrictions can be imposed by ordering real variables before the monetary policy variable in the Choleski decomposition of the long-run relations among the variables. The ordering used in this paper is *LRGDP*, *LRPCOM*, *FFR*, *NBR*, *TR*, and *LPCOM*.[6]

Empirical evaluation of alternative lag structures: Monte Carlo simulations

A fundamental problem in choosing between a symmetric or an asymmetric lag structure in empirical applications of VARs is that the true lag structure is not known. The aim of the Monte Carlo experiment is to determine whether there are significant differences in the estimated effects of monetary policy shocks from a VAR estimated with symmetric (asymmetric) lags when the true lag is asymmetric (symmetric). The first step in the Monte Carlo simulation is to specify the true lag structure of the VAR and then assign values for the coefficients in the VAR and values for the variance-covariance matrix for the VAR. This then allows computation of the true impulse response function. For concreteness, assume that the true lag structure is specified to be symmetric. For each of the 500 draws in the simulation, artificial series for the variables in the VAR are generated, and a statistical criterion is used to specify an asymmetric lag structure. Impulse response functions are then computed. The mean impulse response function across all draws is computed and is plotted along with the true impulse response function. A *t*-test of whether the mean error in the estimated impulse response function across the draws is zero is then performed. This process is repeated assuming that the true lag structure is asymmetric.

To illustrate the process in more detail, consider a structural model with N variables which follows the true data generating process:

$$\Phi_0 y_t = C + \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + v_t \quad (4)$$

where Φ_0 is the contemporaneous coefficient matrix (which has ones on the diagonal and may have non-zero elements on the off-diagonal), v_t is a $N \times 1$ vector of structural errors, which are identified using one of the four methods outlined earlier, with covariance matrix $\sigma^2 I$, C is a $N \times 1$ vector of constants, and Φ_i is an $N \times N$ coefficient matrix. By premultiplying both sides by Φ_0^{-1}, we obtain the VAR representation.

$$y_t = \Phi_0^{-1} C + \Phi_0^{-1}\Phi_1 y_{t-1} + \cdots + \Phi_0^{-1}\Phi_p y_{t-p} + \Phi_0^{-1} v_t \quad (5)$$

For convenience, we can rewrite Equation 5 as

$$y_t = D + \beta_1 y_{t-1} + \cdots + \beta_{t-p} y_{t-p} + e_t \quad (6)$$

where D is $\Phi_0^{-1}C$, β_i is a reduced-form coefficient matrix which equals $\Phi_0^{-1}\Phi_i$, and e_t is a vector of VAR residuals, i.e. $\Phi_0^{-1} v_t$, with variance-covariance matrix $\Sigma(=\sigma^2\Phi_0^{-1}\Phi_0'^{-1})$. Consequently, y_t can be generated using Equation 6 by randomly drawing values for e_t from $N(0,\ \sigma^2\Phi_0^{-1}\Phi_0'^{-1})$. However, before the y_t can be generated, values for the matrices, β_i, and the variance-covariance matrix of e_t, Σ, and the lag length need to be specified.

In the spirit of Kennedy and Simons (1991), the parameter settings (namely the β_i matrices) and the variance-covariance matrix Σ of the random errors for the simulations were obtained from estimation of symmetric and asymmetric

[6] The following assumptions are made to identify monetary policy shocks: (1) shocks to monetary policy have no long-run effects on output; (2) shocks to monetary policy have no long-run effects on the relative price of commodities; and (3) shocks to monetary policy have no long-run effects on the interest rate. The first and the third restrictions are familiar results of the IS-LM aggregate demand–aggregate supply model. A positive shock to monetary policy initially raises output above the natural level by raising real money balances which shifts the LM curve and the aggregate demand curve. Consequently, as we move up the positively sloped short-run aggregate supply curve, output rises above the natural level. The interest rate falls initially since real balances have risen. However, in long-run equilibrium, as prices adjust and we return to the natural level of output, real money balances return to their initial level as do output and the interest rate. The second restriction is another aspect of the assumption of neutrality. That is, monetary policy has no effect on relative prices in the long run. Note that no restrictions are placed on the effect of monetary policy shocks on total reserves, commodity prices, or the overall price level in the long run. Thus monetary policy shocks are allowed to alter total reserves in the long run. No long-run effects on real commodity prices in conjunction with long-run effects on commodity prices implies that monetary policy shocks have long-run effects on the overall price level that are the same magnitude as the long-run effects on commodity prices. Other implications of this ordering are discussed in McMillin (2001).

Table 1. *Selected lag lengths for Keating-type asymmetric lag VARs*

(a) CEE, STR, and BM identification schemes						
	LRGDP	*LGDPD*	*LPCOM*	*NBR*	*TR*	*FFR*
AIC	7	2	6	5	3	2
(b) Long-run restrictions approach						
	DLRGDP	*DLRPCOM*	*DFFR*	*DNBR*	*DTR*	*DLPCOM*
AIC	1	3	5	1	1	6

Notes:
LRGDP: log of real gdp.
LGDPD: log of gdp deflator.
LPCOM: log of the commodity price index.
NBR: normalized nonborrowed reserves plus extended credit adjusted for reserve requirement changes.
TR: normalized total reserves adjusted for reserve requirement changes.
FFR: the federal funds rate.
DLRGDP: first difference of log of real gdp.
DLRPCOM: first difference of (log of commodity prices – log of gdp deflator).
DFFR: first difference of the federal funds rate.
DNBR: first difference of *NBR*.
DTR: first difference of total reserves, and
DLPCOM: first difference of log of commodity prices.

models using quarterly data for the period 1962:1–1997:4.[7] Data from 1962:1–1964:4 are used as pre-sample data since the reserve measures are constructed using a 12-quarter moving average. The models are estimated over the period 1965:1–1997:4.

Following Christiano *et al.* (1996), a lag of four quarters for each variable in each equation is used for the symmetric lag structure. For the asymmetric lag structure, a systematic search process is employed to determine the appropriate lag. Specifically, Akaike's information criterion (AIC) is used to determine the lag on a variable. Recall that the lag length is allowed to differ across variables, but is the same for a particular variable in each equation of the model. As usual, the lag structure that generates the minimum AIC is selected as the optimal structure. We note that the search process involves significant computational costs in terms of time; hence, a maximum of eight lags was considered.[8] Schwarz's information criterion (SIC) was also used, but Ljung–Box Q-tests indicated that residuals from the model using the SIC lag structure were characterized by severe serial correlation.[9] There were no serial correlation problems for the models estimated using the AIC lag structure. Consequently, Table 1 presents only the AIC lag structures.

Once the lag length, parameter values, and variance-covariance matrix for the true model were specified, values for the e_t were selected as random draws from a normal distribution with mean zero and variance-covariance matrix equal to the estimated variance-covariance matrix, and simulated series for y_t were constructed using Equation 6. For each of the 500 draws of the simulation, 632 observations were generated in this fashion. However, in order to allow the simulated y_t series to settle down, the first 500 observations were discarded; only the last 132 observations (the length of the period 1965:1–1997:4) were used for the estimation of the impulse response functions.

Once the simulated series were generated for a particular draw, they were used to specify the lag length and estimate impulse response functions. For example, using the simulated series and assuming the symmetric lag structure with four lags was the true lag structure, the search process described earlier was used to determine the optimal lag

[7] Quarterly data rather than monthly data are used because of the time required to perform the Monte Carlo simulations described later in the paper. Using monthly data renders these simulations infeasible. Impulse response functions estimated using quarterly data are similar in pattern and magnitude to those estimated using monthly data.

[8] When the number of lags for the six variable model ranges from 1–8, there are 262,144 ($=8^6$) possible asymmetric lag VAR specifications. In this case, using a Pentium III processor, it took approximately 1.5 hours to complete the search. The computation time becomes a serious problem in the Monte Carlo simulations since the lag length must be re-specified for each draw of the simulation.

[9] The AIC and SIC are defined as:

$$\text{AIC} = T\log|\Sigma| + 2N$$
$$\text{SIC} = T\log|\Sigma| + N\log(T)$$

where $|\Sigma|$ is the determinant of the variance-covariance matrix of the residuals, N is total number of parameter estimates in all equations, and T is number of usable observations.

Table 2. *Percentage of time lag length selected*

	Keating-type asymmetric lag search (AIC)											
	Variable 1		Variable 2		Variable 3		Variable 4		Variable 5		Variable 6	
Lag	CR	LR	CR	LR	CR	LR	CR	LR	CR	LR	CR	LR
1	2.0	13.6	0.6	0.0	0.8	6.6	0.4	0.0	0.6	4.4	0.8	0.0
2	9.2	20.2	7.8	0.0	6.2	13.4	15.4	13.2	9.8	19.8	22.6	0.0
3	34.6	4.6	11.2	4.4	11.4	20.2	24.0	6.6	15.6	4.4	36.0	4.4
4	33.2	47.6	57.6	68.6	53.0	45.2	39.4	50.8	49.4	44.0	22.0	75.0
5	9.6	7.2	11.4	11.0	13.8	7.2	9.4	14.0	12.2	14.0	8.4	4.4
6	5.2	2.4	4.0	9.4	6.8	2.6	4.2	8.8	6.4	8.8	4.6	14.0
7	3.4	2.2	4.8	0.0	4.4	2.2	4.0	0.0	2.4	4.6	3.2	2.2
8	2.8	2.2	2.6	9.6	3.6	2.6	3.2	6.6	3.6	0.0	2.4	0.0
Mean	3.8	3.4	4.2	4.5	4.3	3.6	3.9	4.3	4.1	3.9	3.5	4.3

Note: CR denotes the contemporaneous restriction schemes while LR represents long-run restrictions scheme. In the column labelled CR, variables 1, 2, 3, 4, 5, and 6 correspond to output, the price level, commodity prices, total reserves, nonborrowed reserves, and the federal funds rate, respectively. In the column labelled LR, variables 1, 2, 3, 4, 5, and 6 correspond to the first differences of output, the relative price level, the federal funds rate, nonborrowed reserves, total reserves, and commodity prices.

structure for the asymmetric lag VAR, and impulse response functions for shocks to the monetary policy variable were then estimated. This was done for each of the 500 draws in the simulation. For the artificial series generated when the asymmetric lag structure was assumed to be the true lag structure, a symmetric lag of four quarters was used in estimating the impulse response functions. A relatively small number of replications, 500, was chosen for the simulation because of computing time limitations. As noted in footnote 8, the asymmetric lag search process for a six variable system with a maximum lag of 8 required about 1.5 hours to finish an iteration using a PC with Pentium III processor.

The effect of lag structure misspecification on the impulse responses was evaluated using two approaches. First, to provide convenient visual comparison, the mean of the point estimates of the impulse response functions from the 500 draws for the misspecified models was plotted along with the point estimates from the true model. Next, a formal test of the hypothesis that the differences between the true point estimates and the point estimates from the alternative lag VAR are zero was computed. That is, the mean-error (me) for the difference between the true impulse response functions and the estimated impulse response functions was computed, and *t*-statistics under the null hypothesis that the mean-error = 0 were calculated and compared to critical values.[10]

III. EMPIRICAL RESULTS FROM THE MONTE CARLO SIMULATIONS

Simulation I: true lag structure is symmetric

This section investigates the effects of specifying an asymmetric lag structure using the AIC criterion when the true lag structure is a symmetric structure with four lags. Before the results for the impulse response functions are presented, the results of the asymmetric lag selection process are summarized. Table 2 presents the percentage of the 500 draws that selected a particular lag length for each variable. For each of the six variables, there is a column labelled CR for the models that use contemporaneous restrictions to identify monetary policy shocks and an analogous column labelled LR for the model that uses long-run restrictions. Recall that the contemporaneous restrictions schemes use a model in which all variables are in log levels or levels while the long-run restrictions scheme is based upon a model in which the variables are in first differences.[11]

For the contemporaneous restrictions schemes, the lag lengths selected for each variable tend to cluster in lags three, four, and five. For example, for the first variable, the true lag length, four, is selected 33% of the time, while three lags are specified 35% of the time. For the second, third, fourth, and fifth variables, the true lag length is selected 58%, 53%, 39%, and 49% of the times. However, for the sixth variable, four lags are selected

[10] Specifically, the mean-error of the impulse response (irf) for horizon h is defined as:

$$me_h = \frac{1}{R}\sum_{i=1}^{R} (irf_h - trueirf_h)$$

where $h = 0, 1, \ldots, 15$ and R is the number of replications, i.e. 500.

[11] For the contemporaneous restrictions models, the first through sixth variables correspond to output, the price level, commodity prices, total reserves, nonborrowed reserves, and the federal funds rate, respectively. As noted earlier, for the long-run restrictions scheme, the model is slightly modified. The first through sixth variables for the long-run restrictions scheme correspond to the first differences in output, real commodity prices, the federal funds rate, nonborrowed reserves, total reserves, and commodity prices, respectively.

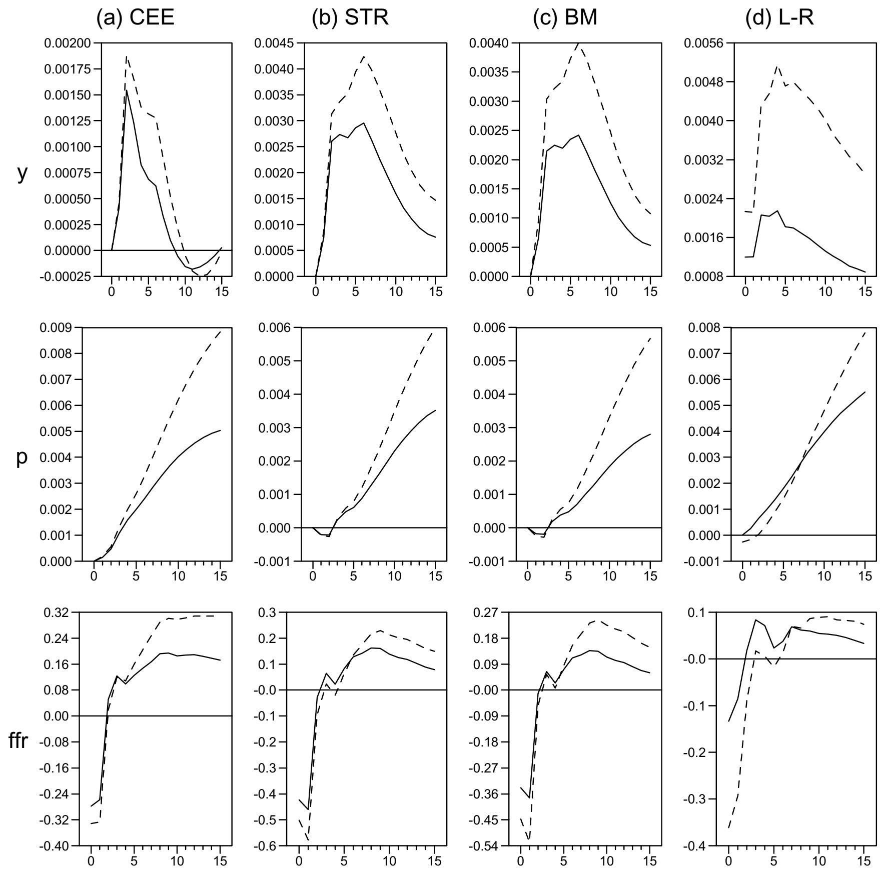

Fig. 1. *Impulse response functions: true lag is symmetric*
Note: *The dotted lines are the true impulse response functions from the asymmetric lag VARs while the solid lines are the impulse response functions from the misspecified models with symmetric lag structure.*

only 22% of the time. For this variable, three lags are specified 36% of the time, and two lags are selected 23% of the time. Finally, the mean of the specified lag length for each variable ranges from 3.5 to 4.3. The mean of the specified lag length across all variables is slightly less than 4; the mean is 3.8 (not reported in Table 2).[12]

For the long-run restrictions scheme, four lags are selected more frequently than for the contemporaneous restrictions schemes. For the first variable (the first difference of the log of output), the true lag length, four, is selected 48% of the time. Also, for the second, third, fourth, fifth, and sixth variable, this lag length is selected 69%, 45%, 51%, 44% and 75% of the time. However, the means of the specified lag length for each variable range from 3.4–4.5, and this range is very similar to the range for the contemporaneous restrictions schemes.

The effects of the lag structure misspecification on impulse response functions are presented in Fig. 1. This figure graphs the true impulse responses for output, the price level, and the federal funds rate as well as the mean impulse response function from the asymmetric lag structure models that are estimated on each draw. The first column of this figure presents the results for the CEE scheme. The remaining columns present analogous results for the STR, BM, and LR schemes, respectively. In each diagram, the solid line is the mean of the point estimates for the asymmetric lag VARs while the dotted line represents the point estimates from the true model.

Several points are worth noting. The pattern of effects is very similar and the largest effects occur at basically the same point in time. However, the magnitude of effects is quite different, especially after the first four or five quarters. The impulse response functions from the asymmetric lag models generally indicate effects that are weaker than the true effects. For the contemporaneous restrictions schemes, the effects become noticeably weaker after four or five quarters for output and price. For the federal funds rate, the impact effect is weaker, but the effect is approximately the same as the true effect for quarters three to five, and then is weaker than the true effect after that. For the long-run restrictions approach, the effects on output indicated by the impulse response function from the asym-

[12] However, no case in the 500 replications correctly selected 4 lags for all six variables.

metric lag model are substantially weaker than the true effects at virtually all horizons. The initial effects on the federal funds rate are also substantially weaker than the true effects. For price, the effects are initially somewhat stronger than the true effects, but then become noticeably weaker at longer horizons. The difference between the asymmetric lag impulse response function and the true impulse response function for output for the long-run restrictions scheme is greater than for the contemporaneous restrictions schemes. The same is true for the initial effects on the federal funds rate.

As noted earlier, the question of whether these differences are significant are examined using formal test statistics; mean-errors between the estimated impulse response functions and the true impulse response functions across the 500 replications are calculated, and t-statistics are used to test the null hypothesis that the mean-error $= 0$ against the alternative hypothesis that the mean-error $\neq 0$ for each horizon. However, in order to conserve space, only the results for horizons 1, 3, 5, 7, 9, 11, 13, and 15 are reported. The calculated mean-errors and their standard errors are presented in Table 3. In the table, panels

Table 3. *Impulse response functions mean-errors (me). True lag structure: symmetric lag*

	Output		Price Level		Federal Funds Rate	
	me($\times 10^{-4}$)	se($\times 10^{-4}$)	me($\times 10^{-4}$)	se($\times 10^{-4}$)	me($\times 10^{-1}$)	se($\times 10^{-1}$)
Panel A: CEE						
Horizon						
1	−0.471	0.274[c]	−0.361	0.085[a]	0.684	0.044[a]
3	−4.213	0.478[a]	−2.382	0.204[a]	0.048	0.061
5	−6.288	0.544[a]	−6.116	0.341[a]	−0.344	0.070[a]
7	−5.637	0.568[a]	−11.742	0.514[a]	−0.804	0.069[a]
9	−2.595	0.600[a]	−18.481	0.707[a]	−1.070	0.067[a]
11	0.168	0.642	−25.497	0.886[a]	−1.151	0.066[a]
13	1.146	0.668[c]	−32.093	1.040[a]	−1.240	0.065[a]
15	0.361	0.677	−37.934	1.170[a]	−1.374	0.064[a]
Panel B: STR						
Horizon						
1	−1.139	0.271[a]	0.217	0.076[a]	1.176	0.041[a]
3	−6.302	0.443[a]	−0.403	0.175[b]	0.414	0.058[a]
5	−10.733	0.496[a]	−1.834	0.288[a]	0.185	0.063[a]
7	−13.509	0.519[a]	−4.990	0.431[a]	−0.334	0.059[a]
9	−12.689	0.536[a]	−9.350	0.589[a]	−0.687	0.052[a]
11	−10.416	0.549[a]	−14.513	0.666[a]	−0.765	0.048[a]
13	−8.336	0.545[a]	−19.851	0.802[a]	−0.749	0.045[a]
15	−7.128	0.527[a]	−24.773	0.916[a]	−0.704	0.045[a]
Panel C: BM						
Horizon						
1	−3.030	0.312[a]	0.630	0.085[a]	1.529	0.104[a]
3	−9.688	0.592[a]	−0.627	0.191[a]	0.094	0.074
5	−13.795	0.702[a]	−2.678	0.317[a]	−0.133	0.070[c]
7	−15.187	0.734[a]	−6.623	0.496[a]	−0.724	0.067[a]
9	−13.700	0.725[a]	−11.855	0.704[a]	−1.085	0.060[a]
11	−10.226	0.731[a]	−17.761	0.899[a]	−1.097	0.053[a]
13	−7.241	0.740[a]	−23.568	1.064[a]	−1.013	0.048[a]
15	−5.428	0.735[a]	−28.668	1.196[a]	−0.881	0.048[a]
Panel D: LR						
Horizon						
1	−9.100	0.814[a]	4.241	0.853[a]	2.076	0.109[a]
3	−25.204	0.888[a]	5.809	1.057[a]	0.661	0.096[a]
5	−28.879	0.951[a]	4.065	1.328[a]	0.414	0.073[a]
7	−29.296	0.889[a]	0.467	1.527	−0.013	0.064
9	−28.147	0.813[a]	−5.433	1.740	−0.267	0.050[a]
11	−25.008	0.694[a]	−11.005	1.953[a]	−0.374	0.042[a]
13	−22.558	0.605[a]	−17.213	2.144[a]	−0.368	0.035[a]
15	−20.107	0.522[a]	−22.797	2.317[a]	−0.409	0.029[a]

Note: Panels A, B, C, and D display the impulse response function mean-error (me) and its standard error (se) for the CEE, STR, BM, and LR schemes.
[a] Significant at 1% level.
[b] Significant at 5% level.
[c] Significant at 10% level.

A, B, C, and D present the results for the asymmetric lag VAR in which monetary policy shocks are identified using the CEE, STR, BM, and LR schemes, respectively.

The results indicate that, for all identification schemes and almost all most horizons, the point estimates from the asymmetric lag VARs are significantly different from the assumed true point estimates. The responses are generally weaker, and the differences are substantial for most horizons for output, the price level, and the federal funds rate.

Simulation II: true lag structure is asymmetric

This section investigates the effects of using a symmetric lag structure with four lags when the true lag structure is an asymmetric structure. A lag of four quarters was chosen following Christiano *et al*. (1996), and is commonly used with quarterly data. The asymmetric lag structures used are those reported in Table 1. As in the previous section, the mean of the point estimates of the impulse response functions for the symmetric lag model along with the point estimates from the true asymmetric lag VAR are plotted. Second, the mean-errors between the impulse responses from the true model and from the misspecified model over 500 replications for each horizon are computed. As before, *t*-statistics are used to test whether the mean-errors are significantly different from zero.

Figure 2 plots the mean of the point estimates of the impulse response functions from the symmetric lag VARs (solid lines) along with the point estimates of the true asymmetric lag VAR (dotted lines). Overall, the point estimates from the symmetric lag VAR(4) are different from the true model, although for the contemporaneous restrictions identification schemes, the differences are not large at very short horizons. However, the differences become larger at longer horizons. For the LR scheme, the differences are substantial even at short horizons. As in Fig. 1, the mean impulse response functions from the VARs with

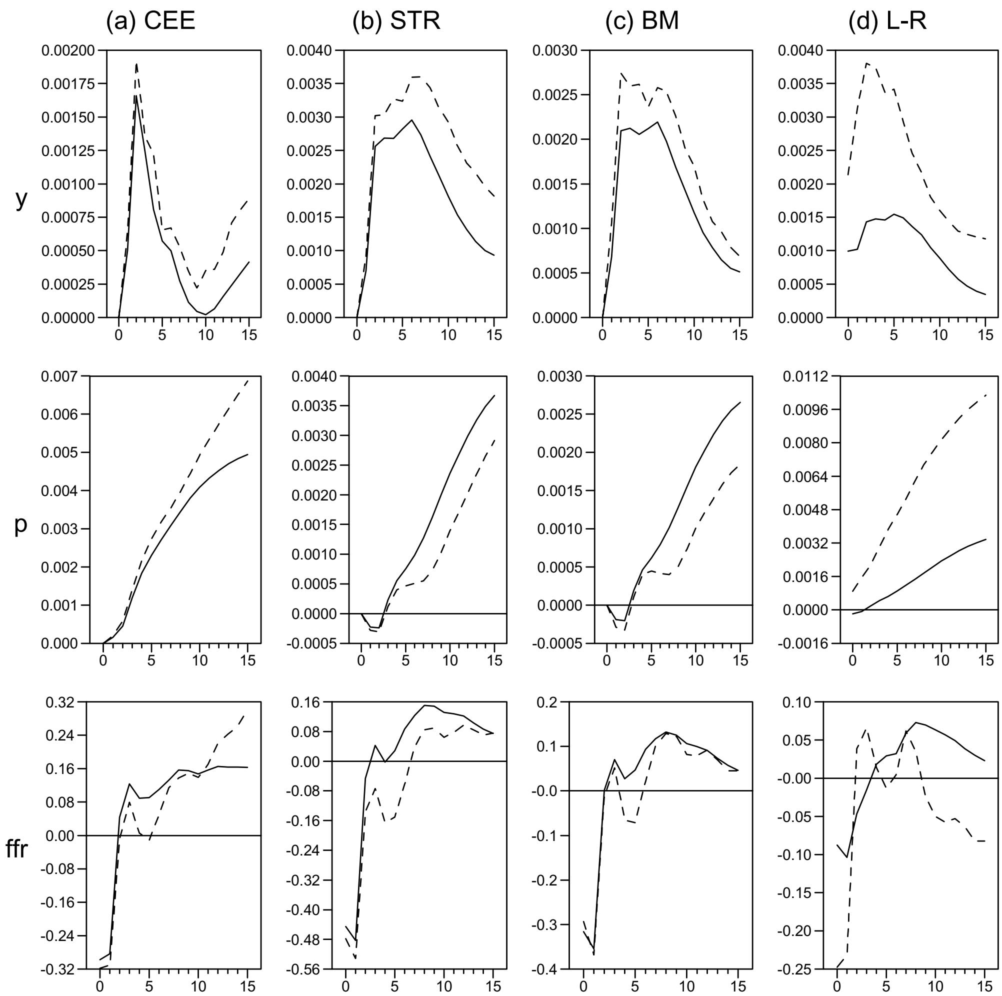

Fig. 2. *Impulse response functions: true lag is asymmetric*
Note: The dotted lines are the true impulse response functions from the asymmetric lag VARs while the solid lines are the impulse response functions from the misspecified models with symmetric lag structure.

Table 4. *Impulse response function mean-errors (me). True lag structure: asymmetric lag*

	Output		Price Level		Federal Funds Rate	
	me($\times 10^{-4}$)	se($\times 10^{-4}$)	me($\times 10^{-4}$)	se($\times 10^{-4}$)	me($\times 10^{-1}$)	se($\times 10^{-1}$)
Panel A: CEE						
Horizon						
1	−1.424	0.251[a]	−0.415	0.091[a]	0.274	0.047[a]
3	−1.116	0.455[a]	−2.071	0.211[a]	0.438	0.060[a]
5	−0.849	0.529	−4.365	0.340[a]	1.024	0.066[a]
7	−2.550	0.562[a]	−4.679	0.490[a]	0.181	0.066[a]
9	−1.753	0.607[a]	−6.379	0.667[a]	0.062	0.063
11	−2.942	0.636[a]	−10.183	0.841[a]	−0.138	0.061[b]
13	−4.647	0.631[a]	−14.425	0.996[a]	−0.782	0.060[a]
15	−4.738	0.608[a]	−19.236	1.129[a]	−1.374	0.060[a]
Panel B: STR						
Horizon						
1	−2.236	0.247[a]	0.582	0.080[a]	0.484	0.042[a]
3	−3.487	0.417[a]	1.100	0.185[a]	1.158	0.057[a]
5	−4.166	0.487[a]	2.811	0.292[a]	1.785	0.058[a]
7	−8.665	0.477[a]	7.220	0.425[a]	0.856	0.056[a]
9	−10.111	0.495[a]	9.683	0.580[a]	0.586	0.052[a]
11	−10.325	0.502[a]	9.707	0.732[a]	0.509	0.046[a]
13	−10.214	0.487[a]	9.087	0.866[a]	0.187	0.044[a]
15	−8.860	0.452[a]	7.550	0.978[a]	−0.005	0.043[a]
Panel C: BM						
Horizon						
1	−3.791	0.311[a]	1.012	0.085[a]	0.136	0.118
3	−4.716	0.544[a]	0.818	0.190[a]	0.182	0.099[c]
5	−2.440	0.735[a]	1.728	0.306[a]	1.185	0.083[a]
7	−5.582	0.869[a]	6.139	0.458[a]	0.152	0.072[b]
9	−4.757	0.957[a]	8.163	0.644[a]	0.029	0.059
11	−3.685	0.960[a]	8.177	0.840[a]	0.203	0.050[a]
13	−3.128	0.908[a]	8.316	1.034[a]	0.033	0.048
15	−1.734	0.825[a]	8.058	1.210[a]	0.013	0.049
Panel D: LR						
Horizon						
1	−21.070	0.883[a]	−1.628	0.035[a]	1.280	0.101[a]
3	−22.653	0.861[a]	−2.553	0.055[a]	−0.808	0.092[a]
5	−18.690	0.751[a]	−3.668	0.081[a]	0.428	0.076[a]
7	−10.974	0.621[a]	4.707	0.104[a]	−0.028	0.064
9	−7.550	0.536[a]	−5.510	0.122[a]	0.940	0.048[a]
11	−7.189	0.458[a]	−6.089	0.137[a]	1.143	0.037[a]
13	−7.773	0.391[a]	−6.548	0.147[a]	1.019	0.030[a]
15	−8.327	0.336[a]	−6.903	0.155[a]	1.052	0.026[a]

Note: see notes to Table 3.

the misspecified lag lengths underestimate the true effect on output. However, for the price level, the true effect is underestimated for the CEE and LR schemes, but is overestimated for the STR and BM schemes. This differs from the previous experiment in which all identification schemes generated weaker effects for price for the VARs with misspecified lag length than the true effects. For the federal funds rate, the estimates from the STR and BM schemes are frequently larger than the true effects while the estimated effects are sometimes larger and sometimes smaller than the true effects for the CEE and LR schemes.

As in the previous section, in order to examine whether the differences between the estimated and true impulse response functions are significant, the mean-errors and *t*-statistics are computed and are presented in Table 4. As before, Panels A, B, C, and D present the results for the CEE, STR, BM, and LR schemes, respectively. In general, the mean-errors are significantly different from zero for all identification schemes. This implies that the distortions in the impulse responses are not trivial when a VAR model is fitted using a symmetric lag structure when the true lag structure is asymmetric.

Discussion

The results of the simulations indicate significant quantitative differences in the estimated impulse response functions when an inappropriate lag structure is employed in the estimation of the effects of monetary policy shocks. This is true for all identification schemes, but the quantitative differences are larger for the scheme that imposes no contemporaneous restrictions on the relations among the variables—the long-run restrictions scheme—than for the schemes that restrict the contemporaneous interactions among the variables. From Figs 1 and 2, we see that this difference is especially large for the contemporaneous liquidity effect. It seems likely in light of the standard view that the liquidity effect is critical in transmitting the effects of monetary policy to output and price that the big differences in the magnitude of the liquidity effects across lag structures helps explain the big differences in the effects of policy shocks on output and price across lag structures.

It is thus apparent that the lag structure of a VAR model does matter when assessing the effects of monetary policy shocks. Previous studies often recognize uncertainty about the true lags only by examining the sensitivity of results to alternative symmetric lag lengths. However, given that uncertainty about the true lags extends to lag structure as well as lag length, it is important that checks of the robustness of empirical estimates of the effects of monetary policy shocks be extended to consider alternative lag structures as well as lag length within a given structure.

IV. SUMMARY AND CONCLUSIONS

This paper examined the implications of lag structure for estimating the effects of monetary policy shocks in a VAR. A symmetric lag structure in which all variables have the same lag length and an asymmetric lag structure in which the lag length differs across variables but is the same for a particular variable in each equation of the model were examined. Consideration of symmetric versus asymmetric lags is important in light of the fact that the true lag structure is generally not known. Based on previous work that suggests that the estimated effects of monetary policy shocks may differ substantially across identification schemes, four commonly used identification schemes are considered. Three of these schemes use restrictions on the contemporaneous relations among the variables to identify monetary policy shocks, and one uses long-run restrictions across the model variables to achieve identification of policy shocks.

Impulse response functions from symmetric lag and asymmetric lag VARs are compared by considering Monte Carlo simulations in which one type of lag structure is set as the true lag structure and the effects of estimating the other type of lag structure are evaluated. For all identification schemes and at virtually all horizons, it is found that the responses from the VARs with misspecified lag structures are significantly different from the assumed true responses. Although the general patterns of effects from the VARs with misspecified lag structures are similar to the patterns from the true models, policy evaluation requires knowledge of quantitative effects rather than just general patterns. The simulations strongly indicate that the lag structure of a VAR model does matter when assessing the quantitative effects of monetary policy shocks. Given the inherent uncertainty about the true lag structure in practice, it is thus important that one compare the impulse response functions from both symmetric lag and asymmetric lag VARs in assessing the effects of monetary policy shocks.

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