JAMES S. FACKLER
University of Kentucky

W. DOUGLAS McMILLIN
Louisiana State University

*Specification and Stability of the Goldfeld Money Demand Function**

Despite intensive investigation of the temporal stability of the Goldfeld formulation of the money demand function, a clear consensus on its stability has yet to emerge. This paper builds a statistical case supporting the first difference of log-levels specification, as opposed to the more commonly used log-levels specification, of the Goldfeld equation and then examines the stability of both specifications. Formal stability tests proposed by Cooley and Prescott, Farley and Hinich, and Brown, Durbin, and Evans are employed; the out-of-sample predictive performance is examined as well. These tests strongly support the first difference specification over the log-levels specification.

1. Introduction

The temporal stability of the standard Goldfeld (1973) formulation of the narrowly defined money demand function (see Table 1, Specification 1) has been investigated intensively in recent years [see, e.g., Goldfeld (1973, 1976); Boughton (1981); Hafer and Hein (1979, 1980); Porter, Simpson, and Mauskopf (1979); Laumas and Mehra (1976); and Laumas and Spencer (1980) for studies employing this specification or some close variant thereof]. Yet, despite this research activity, a clear consensus on the stability of this function has yet to emerge. In view of the importance of the stability properties of the money demand function to the formulation of optimal monetary policy [see Poole (1970)] and in light of the observation of Judd and Scadding (1982) that the Goldfeld function is "still the received version today," further discussion of the stability issues as they relate to the Goldfeld money demand function is warranted.

Our primary objective is to build a statistical case supporting the first difference of log-levels specification (hereafter, first-difference specification), as opposed to the more commonly used log-levels specification, of the Goldfeld equation and then to investigate the stability properties of the first-difference formulation. We

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TABLE 1. *Summary of Stability Tests Results, (Selected Studies)*

Author	*Specification**	*Stability Test(s) Employed*	*Conclusion*
Goldfeld (1976)	1	Out-of-Sample dynamic simulation	unstable
Hafer and Hein (1979)	1	Out-of-Sample dynamic simulation	unstable
		Brown–Durbin–Evans test	stable
Boughton (1981)	2	Brown–Durbin–Evans test	stable
		Plots of recursive coefficient estimates	unstable
Fackler and Wheeler (1982)	2	Brown–Durbin–Evans test	unstable
Laumas and Mehra (1976)	3	Cooley–Prescott Varying Parameter Model	stable

*The alternative specifications are:

1) $\ln m_t = a_0 + a_1 \ln m_{t-1} + a_2 \ln y_t + a_3 \ln RCP_t + a_4 \ln RTD_t + e_{1,t}$;

2) $\ln m_t = a_0 + a_1 \ln m_{t-1} + a_2 \ln y_t + a_3 \ln RCP_t + a_4 \ln \pi_t + e_{2,t}$;

3) $\ln m_t = a_0 + a_1 \ln m_{t-1} + a_2 \ln y_t^P + a_3 \ln RCP_t + e_{3,t}$;

where m_t = real $M1$ (old) money balances; y_t = real GNP; RCP_t = commercial paper rate; RTD_t = time deposit rate; π = rate of inflation; y_t^P = real permanent income; and $e_{i,t}$, i = 1, 2, 3, are error terms.

are led to a first-difference specification due to econometric problems encountered when the standard Goldfeld equation is estimated in log levels using the Cochrane–Orcutt technique.[1] Then, given a sound statistical basis for choosing the first-difference specification, we investigate the stability properties of this specification and find, in contrast to the log-levels specification, consistent evidence across a wide variety of stability tests supporting the stability of the money demand function.

The paper is organized in the following manner. In Section 2 we review recent studies which provide conflicting evidence on the stability of the log-levels specification of the money demand function. In Section 3, we provide a detailed econometric analysis of the Cochrane–Orcutt estimates of the log-level specification of the Goldfeld money demand function. In the process, we obtain several plausible explanations for the conflicting conclusions regarding the stability of the Goldfeld equation. In Section 4, we report the results of various tests of the stability of the first-difference specification and compare these results with those of the log-level equation. Formal stability tests employed are those proposed by Brown, Durbin, and Evans (1975), Cooley and Prescott (1973a,b; 1976), and Farley and Hinich (1975). Results of informal but widely employed tests, the out-of-sample predictive performance from both static and dynamic simulations, are also reported in this section. Finally, the results of the paper are summarized in Section 5.

2. Review of Previous Studies of the Stability of the Money Demand Function

The stability of the Goldfeld equation or a close variant thereof has been investigated using a variety of techniques, including out-of-sample predictive performance [Goldfeld (1973, 1976)], the Brown–Durbin–Evans cusum-of-squares test [Hafer and Hein (1979), Boughton (1981), Fackler and Wheeler (1982)], and the Cooley–Prescott varying parameter regression technique [Laumas and Mehra (1976)]. As summarized in Table 1, these authors report divergent equations and stability conclusions. Goldfeld, Boughton, and Fackler and Wheeler conclude that the function is unstable while both Hafer and Hein, and Laumas and Mehra conclude in favor of sta-

[1]Recent major studies estimating the Goldfeld money demand function with Cochrane-Orcutt include Goldfeld (1973, 1976) Boughton (1981), Hafer and Hein (1979, 1980), and Porter, Simpson, and Mauskoff (1979).

bility.[2] With the exception of Fackler and Wheeler, the formal stability tests (Brown–Durbin–Evans and Cooley–Prescott) suggest stability while the informal tests (dynamic simulations and coefficient plots) suggest instability. One possible source of these disparities is the fact that different sample periods are used. However, most of the studies employ what may be viewed as an inappropriate coefficient estimator[3] which may also contribute to the divergence of opinion with regard to stability. Conclusions may also differ since some authors estimate modified Goldfeld equations. Finally, and of particular interest to us, is the fact that various authors report quite different estimates of rho.[4]

As Judd and Scadding (1982) note, evidence of instability in the Goldfeld equation has been rationalized by the emergence of money substitutes and rapid financial innovation in the 1970s. These events are asserted to have reduced money demand as economic units have shifted out of currency and demand deposits into new assets like NOW accounts and as firms have employed cash management techniques to reduce demand deposit holdings. However, as Judd and Scadding suggest, there are doubts about the contribution of money substitute growth and financial innovation to the hypothesized instability of the Goldfeld function. For example, the interest-rate ratchet variable introduced by Goldfeld (1976) to capture the effects of financial innovation on money demand did not improve the performance of the equation in dynamic out-of-sample

[2]Cargill and Meyer (1979) employ the systematic parameter evolution model and the state variable model to analyze twenty-four different specifications of money demand. On balance, their results support instability.

In interpreting the Hafer–Hein and Boughton results, it should be noted that the statistical properties of the Brown–Durbin–Evans test in the presence of a lagged dependent variable have not been developed [see Brown, Durbin, and Evans (1975)]. Thus the results of this test are suggestive at best. Furthermore, the Brown–Durbin–Evans test requires the absence of serial correlation and the studies that use this test constrain rho to be a single value over the entire sample period. However, estimates of rho differ greatly over different sample periods and over subsets of a particular sample period (see footnote 4) so that this constraint would appear to be inappropriate.

[3]Specifically, the Cochrane–Orcutt technique does not yield consistent or efficient parameter estimates in the presence of serial correlation and a lagged dependent variable. A technique developed by Hatanaka (1974) does, however, yield consistent and asymptotically efficient parameter estimates. Hatanaka's technique has recently been used by Laumas and Spencer (1980) and Fackler and Wheeler (1982).

[4]The values of rho are 0.35, 0.44, 0.92, and 0.30 for Goldfeld, Hafer and Hein (1955*ii*–1972*iv*), Hafer and Hein (1955*ii*–1979*i*), and Boughton, respectively.

simulations over 1974–1976. Furthermore, the static simulation evidence of Hein (1980) is inconsistent with the financial innovation hypothesis which implies continuing shifts in money demand. Likewise, the log-level demand function with money defined as M1 plus RPs and M1 plus RPs and money market funds estimated by Porter, Simpson, and Mauskopf (1979) overpredicts money demand. Finally, although the same factors that led to financial innovation and the growth of money substitutes in the United States—high interest rates and inflation—existed abroad, evidence from other countries suggests that with the exception of Canada money demand remained stable in these other countries. Based upon these considerations and the diverse results from the stability tests, it appears that the stability and specification of the demand for narrowly defined money balances is still an open question.

3. Estimation of the Standard Goldfeld Equation

Given that the authors of the recent literature invariably estimate short-run money demand functions with the Cochrane–Orcutt estimator, it is instructive to investigate some of the characteristics of the resulting equations. Accordingly, in this section we examine the implications of estimating the Goldfeld money demand function by means of the Cochrane–Orcutt and GLS (Prais–Winsten) techniques. In addition, we also consider the standard errors of regressions estimated over various sample periods as well as a typical residual-sum-of-squares (RSS) surface.

Estimates for a variety of equations are presented in Table 2. All data are quarterly, with the dependent variable being real (old) M1. This concept of money is used since it is the definition under investigation in virtually all the literature on the instability of the money demand function; it was, after all, old M1 which was the policy target at the time of the alleged shifts in money demand in the 1970s. We employed the gross national product deflator to deflate both M1 and nominal GNP. The commercial paper rate used is the rate on 4–6 month paper and the rate paid on time deposits is from the FMP data base.

Each equation, in isolation, appears acceptable on most grounds. However, taken as a group, it is evident that the various estimates of the serial correlation coefficient, rho, vary widely with changes in the sample period and depend upon the estimation method chosen; estimated values of rho either fall in the range between 0.39 and 0.47 or else they exceed 0.9. For 1952*ii*–1973*iv*, the Cochrane–

TABLE 2. *Money-Demand-Function Coefficient Estimates (Gold-*

Sample Period	*Estimation Technique*	*Constant*	*Lagged Money*	*Real GNP*
1952*ii*–1973*iv*	Cochrane–Orcutt	0.682	0.659	0.183
		(3.80)	(9.61)	(5.62)
1952*ii*–1973*iv*	GLS	0.741	0.634	0.195
		(3.99)	(9.06)	(5.91)
1952*ii*–1976*iv*	Cochrane–Orcutt	1.075	0.610	0.162
		(3.00)	(7.83)	(4.93)
1952*ii*–1976*iv*	GLS	1.107	0.604	0.163
		(3.06)	(7.73)	(4.93)
1952*ii*–1978*iv*	Cochrane–Orcutt	−0.057	0.994	0.159
		(0.39)	(28.16)	(1.21)
1952*ii*–1978*iv*	GLS	1.299	0.580	0.154
		(3.30)	(7.24)	(4.57)

[a]Natural logs of all variables are employed in the estimation.
[b]Absolute values of t-statistics are in parentheses.
[c]Data used in this paper were obtained from the following sources: $M1$ and the

Orcutt method yields a rho estimate of 0.39 while the GLS method yields an estimate of 0.43.[5] For 1952*ii*–1976*iv* both the Cochrane–Orcutt and GLS methods select a rho of 0.91. Finally, for 1952*ii*–1978*iv*, the estimates of rho are 0.47 and 0.95 for the Cochrane–Orcutt and GLS methods, respectively.

Of particular interest is the distinction between the Cochrane–Orcutt and GLS estimates for the money-demand function over the period 1952*ii*–1978*iv*. The Cochrane–Orcutt technique, employed in a commonly-used software package, selected a rho value, 0.47, that corresponds to a local minimum of the RSS surface (discussed further, and plotted, below). Further, note that the two different estimation methods imply substantially different point estimates of the speed of adjustment of actual to desired money balances.

[5]The GLS method was implemented by varying rho in increments of 0.01. Rho was then selected as the value that minimized the residual sum of squares. The approximate standard error for rho estimated by the GLS method was computed by the method described by Nelson (1973). The calculated standard error of rho = 0.43 is 0.11. An approximate 95-percent confidence interval can be constructed as 0.43 ± 0.22. Thus the estimate of rho from the Cochrane–Orcutt method falls within the confidence interval for the GLS estimate of rho over 1952*ii*–1973*iv*.

feld Specification)[a,b,c]

Commercial Paper Rate	Time Deposit Rate	R^2	*h*	*SE*	*Rho*	Number of Observations
−0.018	−0.044	0.994	1.32	0.0041	0.39	86
(6.30)	(4.16)					
−0.017	−0.049	0.999	1.60	0.0042	0.43	87
(5.87)	(4.65)					
−0.012	−0.045	0.991	0.08	0.0050	0.91	98
(2.98)	(2.81)					
−0.012	−0.048	0.999	0.08	0.0050	0.91	99
(2.97)	(3.39)					
−0.018	0.008	0.990	0.17	0.0052	0.47	106
(5.27)	(0.126)					
−0.013	−0.048	0.999	0.09	0.0051	0.95	107
(3.18)	(3.14)					

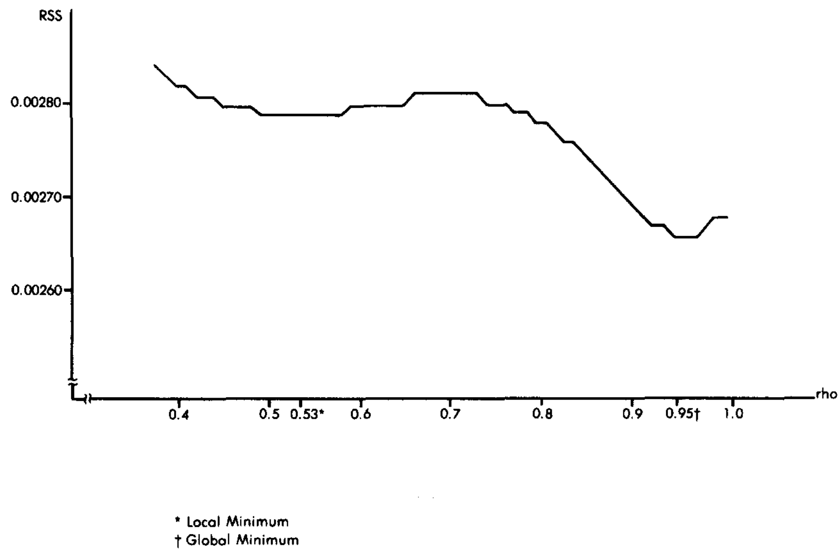
commercial paper (4–6 month) rate: *Federal Reserve Bulletin*, various issues; the time deposit rate: FMP data base; real GNP and the implicit GNP deflator (used to deflate *M*1): *Survey of Current Business*, various issues.

An examination of the RSS surface for the GLS method sheds some light on why the estimated rho changes so dramatically (see Figure 1). The RSS surface is obtained from GLS estimates of the Goldfeld equation in which rho is varied in increments of 0.01. As is evident, the surface is very flat over this range of rho values. A local minimum is found at rho = 0.53 but the global minimum is found at rho = 0.95.[6] However, the total variation in the RSS values from the local minimum to the global minimum is only 0.00011 (4 percent of the value of the residual sum of squares at $\rho = 0.95$). Because of the flatness of this surface, noted in passing by Fackler and Wheeler (1982), it is not only easy to see how a technique such as Cochrane–Orcutt can select a rho corresponding to a local minimum, but also that this technique may lead to misleading parameter estimates.

For our estimates, not only does the value of rho vary across sample periods and estimation techniques but the estimated value

[6]For this period the estimated standard error of rho = 0.53 (which corresponds to the local minimum) is 0.17 and the approximate 95-percent confidence interval is thus 0.53 ± 0.34. The estimate of rho obtained from the Cochrane–Orcutt method is 0.47 and is within the confidence interval for the GLS estimate of rho.

Figure 1.
Residual Sum of Squares of the Goldfeld Money Demand Function, 1952*ii*–1978*iv*



of the RSS also varies across sample periods. For example, for the period 1952*ii*–1973*iv* the RSS is 0.00137 for the Cochrane–Orcutt method and 0.00144 for the GLS method. For the period 1952*i*–1978*iv*, however, the RSS is 0.00270 for the Cochrane–Orcutt method and 0.00266 for the GLS method. Thus the RSS has virtually doubled for both the Cochrane–Orcutt and GLS methods with the addition of five years of data. Furthermore, as implied by the change in the RSS, the value of the standard error of the regression has increased by 25 percent as the sample was extended from 1973*iv* to 1978*iv* (see Table 2).

The dramatic changes in rho, the RSS, and the standard error of the regression as the sample is extended are suggestive of some type of misspecification of the equation. Given an apparent misspecification, search for the proper specification can head in at least two directions. One alternative is to respecify the variables entering the equation, including redefining the dependent variable to account for recent financial innovations. As noted above, there are a number of reasons which may discount the financial innovations explanation of misspecification of the money demand function. A general discussion is included in Judd and Scadding (1982), while articles of specific interest include Hafer and Hein (1979); Enzler, Johnson, and Paulus (1976); and Goldfeld (1973, 1976).[7] As Hafer and Hein (1979) note, however, none of the alternative specifications which they have analyzed is stable over the period 1955*ii*–1977*i* and this is also emphasized by Judd and Scadding (1982).

A second potential source of misspecification is the functional form employed in the estimation. We have already documented that researchers who have used the Cochrane–Orcutt estimation procedure may well have reported parameter estimates which correspond to a local rather than a global minimum of the RSS surface. Indeed, a search technique over values of rho shows that, for the time periods ending in 1976*iv* and 1978*iv*, the RSS is minimized for values of rho in excess of 0.9. Thus, one alternative specification suggested by the data is to employ first-differences of the variables in the estimation.

Aside from the evidence contained in the estimated values of rho supporting first differences, use of this specification is suggested, generally, by the well-known paper of Granger and Newbold (1974). Granger and Newbold argue that a first-difference specification is likely to be useful when the series under analysis are relatively smooth, since in this case the first-order serial correlation coefficient will be near unity. Inspection of the data series employed in this study suggests the existence of significant trends in the data. Thus, estimation of the first-difference of the equation may be a promising alternative.

Despite the argument in the preceding paragraph, it is necessary to proceed cautiously to estimation of first-differences. Granger and Newbold (p. 118) are careful to point out that they "are not advocating first differencing as a universal sure-fire solution to any problem encountered in applied econometric work"; indeed the danger in blithely proceeding to a first-difference specification is carefully developed in Hendry and Mizon (1978), who distinguish between testing for common roots and testing whether common roots, if they exist, are unity. In the Hendry–Mizon analysis, the researcher formally can test for whether a common root of unity exists (in which case differencing is appropriate) rather than impressionistically assuming that, since the data are trending, first differences are appropriate. In contrast, Williams (1978) argues that

[7]The results of estimating and simulating Goldfeld equations for old $M1$ plus *RP*s and money market mutual funds is found in Porter, Simpson, and Mauskopf (1979). These results are very similar to the $M1$ results; demand for these variables is consistently overpredicted for the respecified Goldfeld equations.

first-differencing is an assumption which is not amenable to empirical testing. His argument proceeds by noting that "the *estimated* variance of the error structure will always be finite irrespective of whether or not the *theoretical* variance is finite" (p. 564; emphasis in original). Thus, the Hendry–Mizon technique, which proceeds by transforming a levels equation into a first-difference form and a remainder which includes lagged values of the levels of the variables, must assume that the error structure in the levels form is stationary. Of course, such an assumption cannot be verified with a finite data set.

Thus, while it may not be possible to prove that the first-difference formulation is appropriate, since the results from the grid search over rho strongly suggest such a form, we proceed by estimating first-difference equations. The results from this estimation are presented in Table 3. The coefficients are of the anticipated sign and the h-statistics indicate the absence of serial correlation. We note that these equations differ in a number of important ways from the log-level equations reported in Table 2. First, the constant term was never significant and was dropped from each equation. Second, the first-difference equation coefficients differ substantially from the log-levels coefficients estimated by the Cochrane–Orcutt technique. Third, the standard error (SE) of the first-difference equation rises only 6 percent when the sample is extended from 1973*iv* to 1976*iv* and then to 1978*iv*, in sharp contrast to the GLS log-level equation where the SE jumps by 19 percent when the sample is extended from 1973*iv* to 1976*iv* and 21 percent when the sample is increased from 1973*iv* to 1978*iv*. In light of the differences between the log-levels and first-difference specifications, it should be instructive to examine the stability properties and predictive performances of each. We undertake these tasks in the next section.

4. Stability of the First-Difference and Levels Specifications

In this section we subject the first-difference and levels specifications of the Goldfeld equation to a wide variety of stability tests. Two of these tests, out-of-sample predictive performance from both static and dynamic simulations, are informal, though widely used, stability tests. In addition, the stability properties of the various equations are tested using the Farley–Hinich (1975), the Cooley–Prescott (1973a,b; 1976), and Brown–Durbin–Evans (1973) stability tests. We investigate these tests in turn.

TABLE 3. *Money-Demand-Function Coefficient Estimates (First-Difference Specification)*[a,b]

Sample Period	*Lagged Money*	*Real GNP*	*Commercial Paper Rate*	*Time Deposit Rate*	R^2	h	*SE*
1952*ii*–1973*iv*	0.501	0.188	−0.012	−0.046	0.471	0.084	0.0048
	(5.66)	(3.91)	(3.01)	(2.98)			
1952*ii*–1976*iv*	0.556	0.202	−0.013	−0.049	0.532	−0.322	0.0051
	(7.05)	(4.48)	(3.12)	(2.96)			
1952*ii*–1978*iv*	0.555	0.189	−0.013	−0.047	0.504	−0.350	0.0051
	(7.08)	(4.34)	(3.31)	(2.83)			

[a]The variables in the equation are first differences of natural logarithms.
[b]Absolute values of t-statistics are in parentheses.

Dynamic and Static Simulation Results

One informal method of evaluating the stability of the money demand function that has, in previous studies, suggested instability in the post-1974 period is the evaluation of forecast errors from out-of-sample dynamic simulations of the money demand function. This method has become widely used after Goldfeld's 1973 article. The use of dynamic out-of-sample simulations has recently been criticized by Hein (1980). He suggests that forecasts from static simulations are preferable to dynamic simulation forecasts since dynamic simulations tend to perpetuate once-and-for-all shifts into continuing shifts. Nonetheless, we believe that dynamic simulations provide useful information, in addition to that contained in static simulations, in evaluating the money demand function. This is so since the Federal Reserve, in formulating policy, is interested in forecasts of money demand (as well as other variables) for the next one or two years. The Fed should thus be interested in the dynamic forecasting properties of the money demand function as well as the static forecasting ability of the equation. Based upon these considerations, the summary statistics from both static and dynamid simulations for both the first-difference and log-levels specifications are presented in Table 4. The GLS estimates are used for the log-level simulations.

The summary statistics in Table 4 are based upon forecasts of the log level of real money demand; to obtain comparable forecasts the first-difference and log-levels equations must both be transformed so that the log-level of real money balances is the left hand variable. Thus the simulation results in Table 4 are derived from

$$\begin{aligned}\ln m_t = {} & \hat{a}_0 (1 - \hat{\rho}) + (\hat{a}_1 + \hat{\rho}) \ln m_{t-1} \\ & - (\hat{a}_1 \hat{\rho}) \ln m_{t-2} + \hat{a}_2 (\ln y_t - \hat{\rho} \ln y_{t-1}) \\ & + \hat{a}_3 (\ln RCP_t - \hat{\rho} \ln RCP_{t-1}) \\ & + \hat{a}_4 (\ln RTD_t - \hat{\rho} \ln RTD_{t-1}),\end{aligned}$$

where for first differences, rho is constrained to unity.

As can be seen from Table 4, there are sharp differences between the forecast performances of the first-difference and levels equations. The equation estimated over 1952*ii*–1973*iv* is simulated over 1974*i*–1976*ii* [Goldfeld's (1976) simulation period] and over 1974*i*–1978*iv*. For both the dynamic and static simulations the first-

TABLE 4. *Simulation Results*

		Dynamic Simulations		Static Simulations	
		1st-Difference	Levels	1st-Difference	Levels
Equation Estimated over 1952ii–1973iv					
Out-of-sample simulations over 1974*i*–1976*ii*	Root-mean-square error[a]	0.05	1.27	0.01	0.34
	Root-mean-square % error[b]	0.98%	23.44%	0.21%	6.19%
Out-of-sample simulations over 1974*i*–1978*iv*	Root-mean-square error	0.07	1.47	0.01	0.34
	Root-mean-square % error	1.36%	26.96%	0.16%	6.26%
Equations Estimated over 1952ii–1976iv					
Out-of-sample simulations over 1977*i*–1978*iv*	Root-mean-square error	0.01	7.97	0.01	1.01
	Root-mean-square % error	0.20%	146.38%	0.10%	18.56%

[a]Root-mean-square error $= \sqrt{\frac{1}{n}\sum_{i=1}^{n}(F_i - A_i)^2}$,

where

n = number of periods in the simulation;
F_i = forecast value in i; and
A_i = actual value in i.

[b]Root-mean-square % error $= \sqrt{\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{F_i - A_i}{A_i}\right)^2\right]} \times 100.$

difference equation out-performs the levels equation in terms of both the root-mean-square error and the root-mean-square percent error.[8] The static simulation errors are, as expected, smaller than the dynamic simulation errors. The equation estimated over the period 1952*ii*–1976*iv* is simulated over 1977*i*–1978*iv*. Again, for both the static and dynamic simulations, the first-difference equation out-performs the levels equation. Furthermore, the simulation performance of the levels equation has worsened relative to the equation estimated over the earlier period while the first-difference results are relatively better.[9] Based upon these results, the first-difference specification is preferred to the levels specification.

Farley–Hinich Test Results

The first formal stability test employed is the Farley–Hinich test. In this test, the coefficients that are thought to be unstable are treated as linear functions of time. Thus, if coefficient a_i is thought to be unstable, it is modeled as

$$a_i = a_i^* + \gamma_i t,$$

with $t = 1, \ldots, n$, and where n is the number of observations in the sample. This test adds to the basic equation variables of the form tx where x is a variable whose coefficient is suspected of instability. The coefficients (γ_i) on these variables are then jointly tested for significance from zero; under the null hypothesis that the coefficients are zero an F-test is appropriate. For example, if the coef-

[8]Since the estimated equations have been transformed so that $\ln m_t$ is the left-hand variable, the root-mean-square errors (RMSE) cannot be directly compared to the standard errors of the estimated equations. A priori the transformation made here should increase the RMSE relative to the standard errors; this is observed here.

[9]In terms of the dynamic simulations, the root-mean-square errors (RMSE) and root-mean-square percent errors (RMSE%) are much larger over the ten-quarter simulation period for the equation estimated through 1976*iv* than for that estimated through 1973*iv*. In particular, the forecast error for 1977*i* is very large for the former equation (due primarily to the size of the constant term for the equation estimated through 1976*iv*) and hence is carried forward to subsequent periods, thus accounting for the dynamic simulation RMSE of 7.97 and 146.38%. However, the RMSE and RMSE% for the static simulations are about three times larger than for the earlier period. When the constant term for this equation is suppressed even though it is statistically significant, the RMSE falls to 0.76 (dynamic) and 0.10 (static) and the RMSE% falls to 14.4 (dynamic) and 1.78 (static). In conjunction with the performance of the first-difference equation, these results indicate a preference for the first-difference specification.

ficient on the commercial paper rate is suspected to be unstable, then the equation

$$\ln m_t = a_0 + a_1 \ln m_{t-1} + a_2 \ln y_t + a_3^* \ln RCP_t$$

$$+ \gamma_3(t \ln RCP_t) + a_4 \ln RTD_t + e_t$$

is estimated and the significance of the coefficient γ_3 on the interaction variable ($t \ln RCP_t$) is tested. Since we had no strong prior beliefs as to which coefficients were unstable, each coefficient was separately treated as unstable. Then a joint test of instability was performed. For the first-difference specification, for all these variables and for all sample periods, the null hypotheses that the coefficients on these variables each equaled zero could not be rejected at the 5-percent level. Furthermore, for all sample periods, when these variables were simultaneously added to the first-difference specification the hypothesis that the coefficients jointly equaled zero could not be rejected.[10]

The same variables were added to the levels specification over each sample period. For 1952*ii*–1973*iv* the hypothesis that each coefficient separately equaled zero and the hypothesis that the coefficients jointly equaled zero could not be rejected.[11] For 1952*ii*–1976*iv* the same hypothesis was separately rejected for time, the product of time and real GNP, and the product of time and the time deposit rate at the 5-percent level and for the product of time and the lagged money stock at the 10-percent level. However, a joint test revealed that the hypothesis that these coefficients were jointly equal to zero could not be rejected. For 1952*ii*–1978*iv* results similar to those 1952*ii*–1976*iv* were obtained. The coefficients on time and the products of time with the lagged money stock,

[10]For the period 1952*ii*–1973*iv*, the null hypothesis can be rejected at the 10-percent level for the variable represented by the product of time and the first difference of the natural log of the rate on time deposits. For 1952*ii*–1976*iv*, the null hypotheses can be marginally rejected at the 10-percent level for the coefficients on time itself and on the products of time and the first differences of the natural logs of the lagged money stock and the commercial paper rate. For 1952*ii*–1978*iv* the null hypothesis can be rejected at the 10-percent level for the coefficient on time.

[11]The particular variables added to the levels specification in each period are the products of time and the quasi-difference of the natural log of the explanatory variable, where the quasi-difference is found using $\hat{\rho}$, the GLS estimate of the autocorrelation coefficient.

real GNP and the time deposit rate were found to be significantly different from zero.

Thus over each sample period the test indicated that the first-difference specification is stable. For the levels specification the evidence is mixed. When the Farley–Hinich variables are separately added to the equation instability is found for 1952*ii*–1976*iv* and 1952*ii*–1978*iv*. Joint tests of the coefficients separately found to be unstable, however, suggest stability. These contradictory results for the levels specification are puzzling, though one possibility is implicit in Farley, Hinich and McGuire (1975), who point out that the Farley–Hinich test is robust for gradual shifts in the parameters. It may be that single shifts occurred for some variables or that a more complicated pattern than the linear scheme modeled here occurred over these periods. Rather than model the parameters as quadratic or higher order polynomials in time as Farley, Hinich and McGuire (1975) suggest might be done to test for multiple shifts or erratic drifts, it was decided to bring other formal stability tests to bear on this issue. It is worth emphasizing that both the separate and joint tests for the first-difference specification suggest stability over time.

Cooley–Prescott Estimation Results

The second formal stability test employed here is based upon the varying parameter regression model of Cooley and Prescott (1973a,b; 1976). Within this framework, two types of parameter variation are considered: permanent and transitory, with the permanent component allowing for drift in parameter values over time. Given a model $y_t = \mathbf{x}_t\mathbf{a}_t$, $t = 1, \ldots, n$, where $\mathbf{x}_t$ is a vector of observations on the explanatory variables and $\mathbf{a}_t$ is a conformable vector of coefficients subject to stochastic variation, the sources of coefficient variation are modeled as

$$\mathbf{a}_t = \mathbf{a}_t^p + \boldsymbol{\mu}_t\,,$$

$$\mathbf{a}_t^p = \mathbf{a}_{t-1}^p + \boldsymbol{\nu}_t\,,$$

where $\mathbf{a}_t^p$ is the permanent component of the coefficient vector, $\boldsymbol{\mu}_t$ and $\boldsymbol{\nu}_t$ are independent, normally distributed random variables with mean vector equal to zero and covariance matrices $\mathrm{cov}(\boldsymbol{\mu}_t) = (1 - \gamma)\,\sigma^2\,\boldsymbol{\Sigma}_\mu$ and $\mathrm{cov}(\boldsymbol{\nu}_t) = \gamma\sigma^2\boldsymbol{\Sigma}_\nu$. $\boldsymbol{\Sigma}_\mu$ and $\boldsymbol{\Sigma}_\nu$ are assumed to be known up to some scale factor. The relative magnitude of permanent and transitory changes in the coefficients is reflected by γ, with γ close to 1 indicating that permanent changes are large relative to transitory changes. Under the null hypothesis that $\gamma = 0$ (coefficient vector is not subject to permanent changes), $\hat{\gamma}/\hat{\sigma}(\hat{\gamma})$ is normally distributed with mean equal to zero and variance equal to 1. The symbol $\hat{\gamma}$ is the maximum-likelihood estimate of γ and $\hat{\sigma}(\hat{\gamma})$ the associated standard error.

Because the process generating the coefficients is not stationary, a likelihood function cannot be specified. A likelihood function conditional on the value of the coefficient process at a particular time can, however, be defined. Cooley and Prescott suggest that the most convenient procedure for obtaining the conditional likelihood function is to focus on the value of the coefficient process one period past the sample. Thus in testing stability, as Laumas and Mehra (1976) point out, it is appropriate to reestimate the coefficients by varying the sample period.

In order to apply the Cooley–Prescott model it is necessary to specify Σ_{μ} and Σ_{ν}. Cooley and Prescott (1973a) suggest that, unless one has knowledge to the contrary, the assumption that the relative importance of permanent and transitory changes is the same is reasonable. That is, they argue that the assumption that $\Sigma_{\mu} = \Sigma_{\nu}$ is reasonable and that, unless one suspects that the random changes in the coefficients are correlated, Σ_{μ} and Σ_{ν} can be assumed to be diagonal. Cooley and Prescott also suggest that numerical values for the elements $\Sigma_{\mu} = \Sigma_{\nu} = \Sigma$ can be obtained from the estimates of the equation based upon the assumption of constant coefficients. These suggestions are followed here.

The varying parameter regressions are applied to the first-difference equation with and without a constant and to the levels equations. The estimates of γ, $\sigma(\gamma)$, and Z are reported in Table 5 along with the various specifications of Σ. Cooley and Prescott note that setting the variance of the constant equal to unity is an alternative to the standard assumption of the first-order serial correlation of the error term and is more general since this latter assumption presumes that the effects of all omitted variables decline exponentially at the same rate. Based upon this, the specification of Σ for the levels equation sets the variance of the constant equal to one.

The results presented in Table 5 suggest that the first-difference specification (with and without the constant) is stable while the levels specification is unstable for the periods ending in 1976*iv* and 1978*iv*. As with our previous tests, we again find reason to

TABLE 5. *Cooley–Prescott Estimation Results*

Sample Period	*Covariance Assumption*	*Stability Results*	
	First-Difference Specification without the Intercept[a,b]		
1952*ii*–1973*iv*	Diag Σ = $(0.784E{-}2\ 0.232E{-}2\ 0.2E{-}4\ 0.24E{-}3)$	$\hat{\gamma} = 0.06\ (0.067)$	$Z = 0.90$
1952*ii*–1976*iv*	Diag Σ = $(0.622E{-}2\ 0.203E{-}2\ 0.2E{-}4\ 0.27E{-}3)$	$\hat{\gamma} = 0.016(0.027)$	$Z = 0.59$
1952*ii*–1978*iv*	Diag Σ = $(0.615E{-}2\ 0.19E{-}2\ 0.2E{-}4\ 0.27E{-}3)$	$\hat{\gamma} = 0.011(0.022)$	$Z = 0.50$
	First-Difference Specification with the Intercept[c]		
1952*ii*–1973*iv*	Diag Σ = $(0.1E{-}5\ 0.798E{-}2\ 0.351E{-}2\ 0.2E{-}4\ 0.3E{-}3)$	$\hat{\gamma} = 0.006(0.02)$	$Z = 0.30$
1952*ii*–1976*iv*	Diag Σ = $(0.1E{-}5\ 0.622E{-}2\ 0.297E{-}2\ 0.2E{-}4\ 0.31E{-}3)$	$\hat{\gamma} = 0.004(0.001)$	$Z = 0.36$
1952*ii*–1978*iv*	Diag Σ = $(0.1E{-}5\ 0.12E{-}2\ 0.287E{-}2\ 0.2E{-}4\ 0.31E{-}3)$	$\hat{\gamma} = 0.002(0.007)$	$Z = 0.29$
	Levels Specification[c]		
1952*ii*–1973*iv*	Diag Σ = $(1\ 0.369E{-}2\ 0.824E{-}3\ 0.5E{-}5\ 0.84E{-}4)$	$\hat{\gamma} = 0.002(0.035)$	$Z = 0.06$
1952*ii*–1976*iv*	Diag Σ = $(1\ 0.105E{-}2\ 0.200E{-}3\ 0.7E{-}5\ 0.29E{-}4)$	$\hat{\gamma} = 0.723(0.193)$	$Z = 4.11$
1952*ii*–1978*iv*	Diag Σ = $(1\ 0.813E{-}3\ 0.200E{-}3\ 0.7E{-}5\ 0.25E{-}4)$	$\hat{\gamma} = 0.656(0.211)$	$Z = 3.11$

[a] $Z = \hat{\gamma}/\hat{\sigma}(\hat{\gamma})$ and under the null hypothesis that $\gamma = 0$, Z is distributed normally with mean = 0 and variance = 1. $\hat{\sigma}(\hat{\gamma})$ is given in parentheses following $\hat{\gamma}$.

[b] The order of the variables in the specification of Σ is lagged money, real GNP, commercial paper rate, and time deposit rate.

[c] The order of the variables in Σ is constant, lagged money, real GNP, commercial paper rate, and time deposit rate.

believe that the first-difference equation is stable while the levels specification is not.[12]

Brown–Durbin–Evans Cusum-of-Squares Results

A final formal stability test used in this study is the cusum-of-squares test of Brown, Durbin and Evans (1975). This test involves the calculation of squared one-period forecast errors from recursive regressions. If the regression relation is stable, these forecast errors should cumulate at an approximately constant rate. Stability is tested by computing the cusum-of-squares statistic and comparing the value of this statistic with a critical value. The Brown–Durbin–Evans test was performed for the first-difference specification for the periods 1952*ii*–1973*iv*, 1952*ii*–1976*iv*, and 1952*ii*–1978*iv*, and the cusum-of-squares statistics for these specifications are given in Table 6.

In implementing the test for the log-level specifications, it is necessary to assume that the serial correlation coefficient is constant over the sample period since the Brown–Durbin–Evans test requires the absence of serially correlated errors. However, estimation of the log-levels equation over various subperiods yields divergent estimates of the serial correlation coefficient so that application of the same serial correlation coefficient over the entire

TABLE 6. *Cusum-of-Squares Statistics*

	Period		
	1952ii–1973iv	*1952ii–1976iv*	*1952ii–1978iv*
First-Difference			
without Intercept	0.090	0.155	0.151
First-Difference			
with Intercept	0.087	0.146	0.141
Log-Level			
GLS	0.091	0.178***	0.167***
Cochrane–Orcutt	0.106	0.187**	0.311*

*Significant at 1-percent; **significant at 5-percent level; ***significant at 10-percent level.

[12]An interesting extension of the Cooley–Prescott technique is provided by Swamy, Tinsley, and Moore (1982) who augment the Cooley–Prescott estimator with the Kalman filter. They find substantial volatility in the coefficients in the levels equation through time.

sample period is not strictly appropriate. Furthermore, the properties of this test in the presence of a lagged dependent variable are not known so that the results in Table 6 are at best suggestive. The cusum-of-squares statistics for all periods are not significant for either of the first-difference specifications, thereby leading to nonrejection of the hypothesis of stability. For the GLS estimates, the hypothesis of stability can be rejected at the 10-percent level for 1952*ii*–1976*iv* and 1952*ii*–1978*iv* but cannot be rejected for 1952*ii*–1973*iv*. For the Cochrane–Orcutt estimates, stability can be rejected at the 5-percent and 1-percent levels for 1952*ii*–1976*iv* and 1952*ii*–1978*iv*, respectively, but cannot be rejected for 1952*ii*–1973*iv*. These results suggest a preference for the first-difference specification.

5. Summary and Conclusions

The aim of this paper has been to thoroughly investigate an alternative specification, and the stability over time, of the Goldfeld function of the demand for money. Taken separately, the evidence from each test presented here is suggestive that the first-difference specification is preferred to the log-levels specification. Considered jointly, the evidence presented here strongly suggests a preference for the first-difference specification over the standard Goldfeld specification. In terms of dynamic and static out-of-sample simulation performance the first-difference specification sharply outperforms the log-levels specification. Furthermore, the Farley–Hinich, Brown–Durbin–Evans, and Cooley–Prescott stability tests indicate that the first-difference specification is stable over time while the log-levels specification is temporally unstable. We regard the poor performance of the log-level equation not as an indication of some fundamental instability in the money demand function but rather as evidence of misspecification of the Goldfeld equation, evidence supported by the flat residual sum-of-squares surfaces and the corresponding propensity for the Cochrane–Orcutt technique to converge to a local rather than a global minimum. We do not argue that the first-difference specification employed here is the "best" specification but rather that, given the explanatory variables in the standard Goldfeld equation, the first-difference specification is preferred to the log-level specification. Other research topics within this specification include the investigation of alternative scale variables and alternative rates of return and the effect of possible simultaneity bias on the estimates. For a recent perspective on the simultaneity issue see Cooley and LeRoy (1981). These issues are, however, beyond the scope of this paper.

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