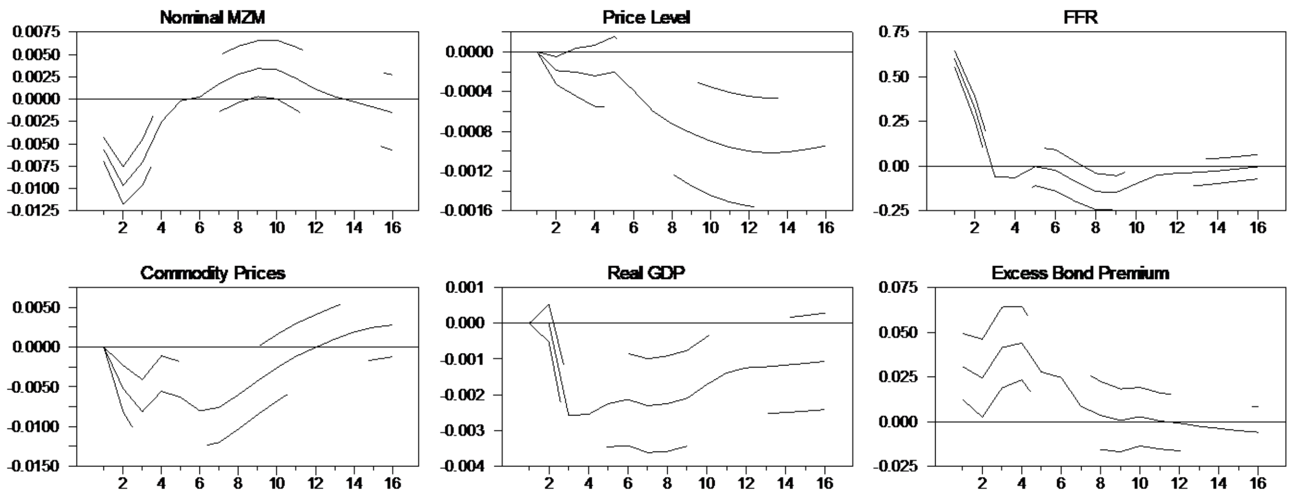
Appendix: Nominal GDP versus Price Level Targeting: An Empirical Evaluation

I: Impulse Response Functions for a Contractionary Monetary Policy Shock

Impulse Responses to a Positive Shock to the Federal Funds Rate



In each panel, the solid line is the point estimate of the impulse response function and the dotted lines are one standard deviation confidence intervals computed using Monte Carlo simulations employing 10,000 draws.

II: Technical Detail of the Methodology

Start with a generic structural model:

$$Y_t = A_0Y_t + A_1Y_{t-1} + \dots A_pY_{t-p} + u_t$$

where the residuals are assumed to be mutually and serially uncorrelated and mean zero. We estimate the reduced form VAR:

$$Y_t = \Pi_1Y_{t-1} + \Pi_2Y_{t-2} + \dots + \Pi_pY_{t-p} + e_t \tag{A1}$$

where $\Pi_i = (I - A_0)^{-1}A_i$ and $e_t = (I - A_0)^{-1}u_t$. As a linear combination of the zero-mean structural shocks, the reduced form residuals are also zero mean. The reduced form coefficient estimates (i.e., the VAR coefficients) can be used to generate dynamic forecasts (base projections or BPs) for the system variables for subsequent periods, conditional on data through period t. Below, we will use the base projections for Y_{t+j} for periods $j=\{1, \dots, m\}$, where m is the policy planning horizon.

Using the lag operator, L, the system can be written as:

$$(I - \Pi_1L + \Pi_2L^2 + \dots \Pi_pL^p)Y_t = e_t$$

and then solved for the moving average representation (MAR):

$$\begin{aligned} Y_t &= (I - \Pi_1L + \Pi_2L^2 + \dots \Pi_pL^p)^{-1}e_t \\ &\equiv C(L)e_t \end{aligned}$$

where C(0) = I. Finally, we can rewrite the MAR in terms of the structural shocks as:

$$Y_t = C(L)(I-A_0)^{-1}(I-A_0)e_t = D(L)u_t$$

where $D(L) = C(L)(I-A_0)^{-1}$ with $D(0) = (I - A_0)^{-1}$ and with the structural shocks $u_t =(I-A_0)e_t$. As is evident from the definition of the lag polynomial D(L), the structural moving average coefficients reflect both the estimated VAR coefficients as well as the coefficients reflecting the contemporaneous links among the variables, the parameters of A_0, which we identify using a Choleski decomposition.

Fundamental to our analysis is the historical decomposition, which in its basic form is found by advancing the MAR by m periods and then decomposing the resulting expression into two terms:

$$Y_{t+m} = \sum_{s=0}^{m-1} D_s u_{t+m-s} + \sum_{s=m}^{\infty} D_s u_{t+m-s} \tag{A2}$$

or

(A3) $$Y_{t+m} = \sum_{s=0}^{m-1} D_s u_{t+m-s} + BP_{t+m}$$

As indicated, the second term on the right side of equation (A3) is the dynamic forecast or base projection (BP) of Y_{t+m} conditional on information at time t and is generated from the reduced form VAR estimation; dynamic forecasts of equation (A1) yield the second term on the right-hand side of equation (A3). In principle, this expression requires shocks over the infinite past. This could in practice be approximated by assuming that shocks prior to the earliest observation assumed their expected values of zero, along with the observation that for a stationary process, the structural coefficients in D_s will converge to the null matrix as *s* becomes arbitrarily large. An alternative approach, which we follow in practice, is to compute the dynamic forecast or base projection directly from the reduced form estimation. For example, for a first-order VAR, $Y_{t+1} = \prod_1 Y_t + e_{t+1}$, estimation yields values for $\prod_1$ so the forecast of Y_{t+1} as of *t* is

$$E(Y_{t+1}) = \prod_1 Y_t$$

since $E(e_{t+1}) = 0$. With $Y_{t+2} = \prod_1 Y_{t+1} + e_{t+2}$, the time *t* forecast of Y_{t+2} is

$$E(Y_{t+2}) = \prod_1 E(Y_{t+1}) = \prod_1 (\prod_1 Y_t) = \prod_1^2 Y_t .$$

By extension, $E(Y_{t+m}) = \prod_1^m Y_t$. Generalization for forecast horizons *m* > 2 and a VAR of order *p*>1 is straightforward.

Our primary focus is on the first term on the right side of equation (A3). This term shows the influence on Y_{t+m} of the shocks to the variables in the system over the planning horizon, periods t+1 through t+m. The elements of this term show how the system would fluctuate around the base projection over the planning horizon, given random disturbances in the economy as characterized by the structural shocks. Even though the expected values of these shocks are zero, policy makers know that the realizations of these shocks are likely to be nonzero. We will proxy the underlying shocks during the planning horizon with random draws from the estimated structural shocks from the VAR model.

Since our implementation is conducted using the MAR, as a simple example analogous to our text discussion of the VAR methodology employed in our counterfactual simulations, consider a two-variable system estimated through period t. With the contemporaneous structural elements identified with the Choleski decomposition and given the structural coefficients, from the MAR we have

$$\begin{bmatrix} y_{1,t+1} \\ y_{2,t+1} \end{bmatrix} = \begin{bmatrix} d_{0,11} & 0 \\ d_{0,21} & d_{0,22} \end{bmatrix} \begin{bmatrix} u_{1,t+1} \\ u_{2,t+1} \end{bmatrix} + \begin{bmatrix} BP_{1,t+1} \\ BP_{2,t+1} \end{bmatrix}$$

Assume that y_1 is the target variable and y_2 is the policy variable. Given the recursive nature of the Choleski decomposition, setting a policy innovation, $u_{2,t+1}$ will have no impact on y_1 in period t+1. Advance the equation by one period:

$$\begin{bmatrix} y_{1,t+2} \\ y_{2,t+2} \end{bmatrix} = \begin{bmatrix} d_{0,11} & 0 \\ d_{0,21} & d_{0,22} \end{bmatrix} \begin{bmatrix} u_{1,t+2} \\ u_{2,t+2} \end{bmatrix} + \begin{bmatrix} d_{1,11} & d_{1,12} \\ d_{1,21} & d_{1,22} \end{bmatrix} \begin{bmatrix} u_{1,t+1} \\ u_{2,t+1} \end{bmatrix} + \begin{bmatrix} BP_{1,t+2} \\ BP_{2,t+2} \end{bmatrix}$$

While the policy innovation in period t+1 does not affect the target in period t+1, it does have an impact in period t+2. That is, the first equation in system (A3) in period t+2 is

$$y_{1,t+2} = d_{0,11}u_{1,t+2} + d_{1,11}u_{1,t+1} + d_{1,12}u_{2,t+1} + BP_{1,t+2}$$

Specifying a target value for $y_{1,t+2} = y^*_{1,t+2}$ and taking random draws from the structural residuals and denoting these with a carat, the above equation can be solved for a value of $u^*_{2,t+1}$ that attains the target value:

$$u^*_{2,t+1} = \frac{1}{d_{1,12}}\{y^*_{1,t+2} - d_{0,11}\hat{u}_{1,t+2} - d_{1,11}\hat{u}_{1,t+1} - BP_{1,t+2}\}$$

Retaining $u^*_{2,t+1}$ as we advance to the next time period allows this policy innovation to be taken into account when the next innovation is computed for the target value for $y_{1,t+3}$, etc. Under the assumption that the shocks to the economy over the policy horizon will be drawn from the same distribution as those during estimation, repeated sampling can provide information about the variability of the target variable around its desired path. Analysis of the policy innovations needed to attain the path for the target variable will reveal, as argued in the text and also below, whether the Lucas critique is operative and whether there is instrument instability. We note that it can be shown that the computed policy innovations described here are identical to those discussed in the text.

Our application builds on the intuition developed in the two-variable examples discussed in MAR form above and in VAR form in the text. Our simulations assume that each period a forward-looking policy maker has a twelve-quarter policy horizon, and Blinder's policy planning process at a given date requires 'an entire hypothetical path' for the policy instrument. To implement this 'first step' of the policy plan for the 'entire hypothetical path,' begin with a random draw of length 2m-1 from the estimated residuals for each equation; with m = 12, the length of the draw covers 23 periods. Assuming these are representative shocks for each equation, for this particular draw at period t and given the shocks to the nonpolicy equations, we need to compute a sequence of policy innovations $\{u^*_{k.t+1}, u^*_{k,t+2}, \ldots, u^*_{k,t+12}\}$. Each policy innovation aims for the desired path for the subsequent 12 quarters, so the policy shock implemented in t+1, $u^*_{k,t+1}$, aims for the path for the target variable for periods {t+1, t+2, …, t+12}. Similarly, the shock $u^*_{k,t+2}$ is implemented with the objective of attaining the path for the target variable for periods {t+2, t+3, …, t+13}, and so on until we finally compute $u^*_{k,t+12}$ with the goal of the target variable path over {t+12, t+13, … , t+23}.

As detailed below, to compute the innovation at period t+1 needed attain the objective over {t+1, t+2, … , t+12}, we take as given not only the shocks to the nonpolicy equations but also the remaining drawn shocks to the policy equation. We note that it is possible for the drawn policy shock for period t+1 to be consistent with the policy objective, in which case this value is retained; otherwise, it is discarded, and the shock needed for the objective is computed. In either case, given the policy innovation $u^*_{k,t+1}$, we next need to select the innovation for period t+2, $u^*_{k,t+2}$, which will attain the policy objective over {t+2, t+3, …, t+13}. Continuing through the process, the final computation at period t is to determine the innovation needed at t+12, given the prior policy innovations, $\{u^*_{k.t+1}, u^*_{k,t+2}, \ldots, u^*_{k,t+11}\}$. This final innovation assures achievement of the objective over {t+12, t+13, …, t+23}. In this manner, for a given random draw from the estimated residuals, we have planned the 'entire hypothetical path' at time t and using this policy path in combination with representative shocks for the nonpolicy equationsand the base projections, we can then compute that trajectory for the system of equations from the MAR.

For a detailed exposition of nominal GDP targeting in our framework, for convenience we place the two variables whose sum we wish to target, say the logs of real GDP and the GDP deflator, as the first and second elements, y_1 and y_2 in the vector Y. The policy variable is thus in position k, $2 < k \leq n$, and the policy shock to this equation is denoted by shock, $u_{k,t+1}$. As above, using a Choleski decomposition, the policy shock in period t+1 cannot y_1 in period t+1. However, it will influence $y_{1,t+2}$, $y_{2,t+2}$, … $y_{1,t+12}$, $y_{2,t+12}$, both directly and indirectly through its impact on other system variables via the system dynamics. Taking as given the values of the system disturbances over the period {t+1, t+2, …, t+12} (holding in reserve the residuals drawn for periods t+13 through t+23), consider the role of $u_{k,t+1}$ on the path of log nominal GDP over periods t+1 through t+12:

$$(y_{1,t+1} + y_{2,t+1}) = (d_{0,11} + d_{0,21})\hat{u}_{1,t+1} + d_{0,22}\hat{u}_{2,t+1} + 0{*}u_{k,t+1} + BP_{1,t+1} + BP_{2,t+1}$$

With a Choleski decomposition, the first term in the MAR, denoted by D(0), is a lower triangular matrix. Thus, the coefficients on all the shocks for $u_{j,t+1}$, j>2, are all zero; here we only explicitly note the zero coefficient on the policy shock, $u_{k,t+1}$. Similarly, highlighting the role of $u_{k,t+1}$ for periods t+2 through t+12:

$$(y_{1,t+2} + y_{2,t+2}) = (d_{0,11} + d_{0,21})\hat{u}_{1,t+2} + d_{0,22}\hat{u}_{2,t+2} + \sum_{i\neq k}^{n}(d_{1,1i} + d_{1,2i})\hat{u}_{i,t+1} + (d_{1,1k} + d_{1,2k})u_{k,t+1} + (BP_{1,t+2} + BP_{2,t+2})$$

$$(y_{1,t+3} + y_{2,t+3}) = (d_{0,11} + d_{0,21})\hat{u}_{1,t+3} + d_{0,22}\hat{u}_{2,t+3} + \sum_{i=1}^{n}(d_{1,1i} + d_{1,2i})\hat{u}_{i,t+2} + \sum_{i\neq k}^{n}(d_{2,1i} + d_{2,2i})\hat{u}_{i,t+1} + (d_{2,1k} + d_{2,2k})u_{k,t+1} + (BP_{1,t+3} + BP_{2,t+3})$$

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$$(y_{1,t+12} + y_{2,t+12}) = (d_{0,11} + d_{0,21})\hat{u}_{1,t+12} + d_{0,22}\hat{u}_{2,t+12} + \sum_{i=1}^{n}(d_{1,1i} + d_{1,2i})\hat{u}_{i,t+11} + \sum_{i=1}^{n}(d_{2,1i} + d_{2,2i})\hat{u}_{i,t+10} + \ldots + \sum_{i=1}^{n}(d_{10,1i} + d_{10,2i})\hat{u}_{i,t+2} + \sum_{i\neq k}^{n}(d_{11,1i} + d_{11,2i})\hat{u}_{i,t+1} + (d_{11,1k} + d_{11,2k})u_{k,t+1} + (BP_{1,t+12} + BP_{2,t+12})$$

In this particular random draw, the value of $u_{k,t+1}$ along with the other disturbances may or may not yield desired values for nominal GDP. The policy objective, of course, is to select a value for the policy shock $u_{k,t+1}$ to attain a desired path for nominal GDP, continuing to hold fixed the values for the other system disturbances. Denote the desired value for nominal GDP in a period t+j as $(y_{1,t+j} + y_{2,t+j})^*$, and substitute these into the above expressions in place of the actual values for j=1,2,..,12. Summing these expressions, on the left side we obtain $\sum_{j=1}^{12}(y_{1,t+j} + y_{2,t+j})^*$ and on the right side we collect terms on $u_{k,t+1}$ and the other shocks and base projections. Conditional on the values for the other shocks, we solve for $u^*_{k,t+1}$, the policy setting needed to attain the target path.[1]

Having found the policy shock for period t+1, update the equations above for periods t+2 through t+13. Solve for the policy shock for period t+2, $u^*_{k,t+2}$, that attains the desired values for nominal GDP conditional on the shock computed above for $u^*_{k,t+1}$, and given the other disturbances for periods 2 through 13. Continue through the policy planning horizon, determining the policy shocks needed to attain the desired values, at each step retaining the previous policy innovations. For a twelve-period planning horizon, then, the last needed shock is for period t+12, computed for the system equations for periods t+12 through t+23. (While a shock for period t+12 has no impact on nominal GDP in t+12 in our setup, it does affect any variables that may be below it in the policy equation. In this case, a complete accounting of the entire system over the planning horizon requires the policy shock for this period.)

The analysis we actually implement modifies the approach above to account for an acceptable tolerance range for the policy process. Generally, if the desired value for nominal GDP in period t+j is $(y_{1,t+j} + y_{2,t+j})^*$, policy makers know it is unrealistic to attain that value exactly. Thus, attaining a value in the range of $(y_{1,t+j} + y_{2,t+j})^* \pm \tau$ is viewed as the actual policy objective. For our computations, if the random draw from the residuals implies that the policy objective is attained for a given period without a

[1] Recall that, consistent with our discussion of Leeper and Zha (2003) above, this computed shock is treated as the policy decision variable, even as it is viewed as random by participants in the economy. Should the drawn shock to the policy innovation be consistent with the policy objective, we continue to view the implied value for the policy variable as a decision by the policy maker.

policy intervention, then computation of the above policy shock for that particular period is not needed; the drawn policy equation residual is just retained. If the drawn system of shocks produce nominal GDP above $(y_{1,t+j} + y_{2,\,t+j})^* + \tau$, we compute the shock needed to return nominal GDP to this upper bound; similarly, if the drawn shocks produced nominal GDP below $(y_{1,t+j} + y_{2,\,t+j})^* - \tau$, we compute a policy shock sufficient to return to this lower bound.[2] Accordingly, the vector of policy shocks over the planning horizon will be a mixture of residuals drawn from the estimation and shocks computed to return nominal GDP to the specified tolerance band if it happens to move outside that band.

Having passed through the data for the simulation period, we combine the policy shocks (some of which may simply be those in the random draw) along with the other shocks for the nonpolicy equations for that particular draw and compute the implied paths of real GDP, the price level, and the other system variables. Finally, the process described above is repeated over 1,000 draws for each so that we can then compute the means and variances of the variables to summarize the statistical properties of the nominal GDP target.

The Leeper and Zha theoretical approach is a Markov-switching model, with each regime a linear model of the economy (a VAR in their case). The effect of a policy intervention is described by the first term on the right side of our equation (1), where our policy interventions are input as the residual of the federal funds rate equation, altering the path of the system variables relative to the base projection.

[2] We select policy to return to the edge of the band for several reasons. First, Brainard (1967) notes that if the policymaker is uncertain about the effect of policy on the economy (multiplicative uncertainty) and uncertain about the direct effect of other factors on the economy (additive uncertainty) and assuming no correlation between these types of uncertainty, the policy response should be in the same direction but less forceful than the indicated policy setting computed under certainty equivalence. While some nonzero values of the correlation between multiplicative and additive uncertainty may overturn this conclusion, Blinder (1997) notes that as a Federal Reserve governor, he nonetheless in practice viewed this “Brainard conservatism principle” as “extremely wise.” Applying this principle to our framework suggests that it would be better for the policy authority to aim at the edge of the tolerance band than at the midpoint of the range. Furthermore, Barlevy (2009) finds that, in the same circumstances as those in Brainard, robust control techniques imply an even more conservative policy response. However, the analysis is more nuanced if there is correlation between multiplicative and additive uncertainty. Second, returning to the edge of the band requires a smaller policy innovation than returning to the midpoint; that is, we undertake the smallest policy action needed to attain the objective. The trade-off is that these smaller interventions may be more frequent than relatively aggressive actions aimed at returning to the midpoint of the band since the probability of a shock moving the economy outside the band is likely higher. Third, there may be a lack of consensus among policy makers on how quickly to approach the target.

Specifically, picking a policy sequence $\{u^*_{k,t+1}, u^*_{k,t+2}, u^*_{k,t+m}\}$, computing the expression $\sum_{s=0}^{m-1} D_s\, u^*_{k,t+m-s}$ and then scaling by $\sqrt{\sum_{s=0}^{m-1} D_s^2}$ provides the "modesty statistic." We note that Leeper and Zha use the *u* shock to the policy equation as the policy innovation and assume as we do that "although the policy advisor chooses [the *u*-innovation], private agents treat it as random" (Leeper and Zha 2003, p. 1678).

Leeper and Zha (2003) argue that the "modesty statistic" has a standard normal distribution, so a computed statistic of less than two implies that the policy innovation embedded in the $\{\hat{u}_k\}$ sequence does not cause agents to alter their assessments about the policy regime in place.[3] We report information on the values of the modesty statistic along with our other results in the text of the paper.

[3] Of course, alternative policy regimes can be "close" to each other, so that distinguishing between these regimes may be difficult. Thus, a modesty statistic of less than 2 is necessary but not sufficient to claim that no important Lucas-critique effects are present.

III: Expanded Table 3

Table 3 (Extended): Loss Functions				
Type Loss Function/ Policy Objective	**% Rate of Change[a]**	**Loss Function Value[*]**		
		Tolerance Band Width		**Continuation Policy[**]**
		±1%	**±2%**	
A. Dual Mandate Weights[b]				
1. Level NGDP	4.5	1.45	1.67	1.76
$(y-y^T)^2$		*0.76 [52%]*	*0.73 [44%]*	
$(p-p^T)^2$		*0.70 [48%]*	*0.94 [56%]*	
2. Level NGDP	5.0	1.99	2.16	2.17
$(y-y^T)^2$		*1.05 [53%]*	*1.19 [55%]*	
$(p-p^T)^2$		*0.94 [47%]*	*0.98 [45%]*	
3. Level NGDP	5.5	2.17	2.80	2.39
$(y-y^T)^2$		*1.02 [47%]*	*1.77 [63%]*	
$(p-p^T)^2$		*1.14 [53%]*	*1.04 [37%]*	
4. Price Level	2.0	3.33	2.18	1.76
$(y-y^T)^2$		*2.83 [85%]*	*1.22 [56%]*	
$(p-p^T)^2$		*0.49 [15%]*	*0.97 [44%]*	
B. Keynesian Weights[c]				
1. Level NGDP	4.5	1.48	1.57	1.67
$(y-y^T)^2$		*1.14 [77%]*	*1.10 [70%]*	
$(p-p^T)^2$		*0.35 [23%]*	*0.47 [30%]*	
2. Level NGDP	5.0	2.04	2.27	2.28
$(y-y^T)^2$		*1.58 [77%]*	*1.78 [78%]*	
$(p-p^T)^2$		*0.47 [23%]*	*0.49 [22%]*	
3. Level NGDP	5.5	2.11	3.17	4.11
$(y-y^T)^2$		*1.53 [73%]*	*2.65 [84%]*	
$(p-p^T)^2$		*0.57 [27%]*	*0.52 [16%]*	
4. Price Level	2.0	4.50	2.31	1.67
$(y-y^T)^2$		*4.25 [95%]*	*1.83 [79%]*	
$(p-p^T)^2$		*1.14 [05%]*	*0.48 [21%]*	

C. Classical Weights[d]				
1. Level NGDP	4.5	1.41	1.77	1.86
$(y-y^T)^2$		*0.38 [27%]*	*0.37 [21%]*	
$(p-p^T)^2$		*1.04 [73%]*	*1.41 [79%]*	
2. Level NGDP	5.0	1.93	2.06	2.06
$(y-y^T)^2$		*0.53 [27%]*	*0.59 [29%]*	
$(p-p^T)^2$		*1.41 [73%]*	*1.47 [71%]*	
3. Level NGDP	5.5	2.22	2.43	2.67
$(y-y^T)^2$		*0.51 [23%]*	*0.88 [36%]*	
$(p-p^T)^2$		*1.71 [77%]*	*1.56 [64%]*	
4. Price Level	2.0	2.15	2.06	1.86
$(y-y^T)^2$		*1.42 [66%]*	*0.61 [30%]*	
$(p-p^T)^2$		*0.74 [34%]*	*1.45 [70%]*	

[*] All values multiplied by e-04. The centered numbers are the total loss function values. The numbers under the total are the weighted squared variability of either y or p around its target. The numbers in brackets are the share of the total loss accounted for by the weighted squared variability of either y or p around its target. The weighted squared variability of y and p around its target may not sum exactly to the total loss due to rounding.

[**] Under the continuation policy, the loss function values for price level targeting with 2% inflation and NGDP targeting with 4.5% growth are the same since these share common trends of 2% growth for prices and 2.5% growth for real GDP around which the MSDs are computed. For Level NGDP targets based on 5.0% and 5.5% growth, the loss functions are based on a price path with 2% growth and real GDP growth of 3.0% and 3.5%, respectively.

[a] The desired rate of change employed in computing the target path of the level of the variable over 2004:1–2006:4. The 2003:4 value is projected forward as the target value at the indicated rate of change.

[b] Dual Mandate Weights: .5 on the variance of both output and the price level from the target value.

[c] Keynesian Weights: .25 on the variance of the price level from target and .75 on the variance of the output from target.

[d] Classical Weights: .75 on the variance of the price level from target and .25 on the variance of the output from target.

Appendix References

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